Laplace Transforms

Definition

$$\mathcal{L}\left[f(t)\right] = \int_0^\infty e^{-st} f(t)dt$$

Common Transforms

$$\mathcal{L}\left[t^n\right] = \frac{n!}{s^{n+1}} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^n}\right] = \frac{1}{(n-1)!}t^{n-1}$$

$$\mathcal{L}\left[e^{\lambda t}\right] = \frac{1}{s-\lambda} \qquad \mathcal{L}^{-1}\left[\frac{1}{s-\lambda}\right] = e^{\lambda t}$$

$$\mathcal{L}\left[\cos(\beta t)\right] = \frac{s}{s^2+\beta^2} \qquad \mathcal{L}^{-1}\left[\frac{s}{s^2+\beta^2}\right] = \cos(\beta t)$$

$$\mathcal{L}\left[\sin(\beta t)\right] = \frac{\beta}{s^2+\beta^2} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^2+\beta^2}\right] = \frac{1}{\beta}\sin(\beta t)$$

Transform of a Derivative

$$\mathcal{L}\left[D^n x\right] = s^n \mathcal{L}\left[x\right] - s^{n-1}x(0) - s^{n-2}x'(0) - \ldots - sx^{(n-2)}(0) - x^{(n-1)}(0)$$

Derivative of a Transform

$$\mathcal{L}\left[t^n f(t)\right] = (-1)^n \frac{d^n}{ds^n}F(s) \qquad \text{where } F(s) = \mathcal{L}\left[f(t)\right]$$

$$\mathcal{L}^{-1}\left[\frac{d^n}{ds^n}F(s)\right] = (-1)^n t^n f(t) \qquad \text{where } f(t) = \mathcal{L}^{-1}\left[F(s)\right]$$

Shift Formula

$$\mathcal{L}\left[e^{\lambda t} f(t)\right] = F(s-\lambda) \qquad \text{where } F(s) = \mathcal{L}\left[f(t)\right]$$

$$\mathcal{L}^{-1}\left[F(s)\right] = e^{\lambda t}\mathcal{L}^{-1}\left[F(s+\lambda)\right]$$

Step and Impulse Functions

$$\mathcal{L}\left[u_a(t)f(t)\right] = e^{-as}\mathcal{L}\left[f(t+a)\right]$$

$$\mathcal{L}^{-1}\left[e^{-as}F(s)\right] = u_a(t)f(t-a) \qquad \text{where } f(t) = \mathcal{L}^{-1}\left[F(s)\right]$$

$$\mathcal{L}\left[\delta(t-a)f(t)\right] = e^{-as}f(a)$$