MATH 2260 (Ordinary Differential Equations I) — Fall 2014
Homework #6

Due Date: Tuesday, November 4, in class or in marking box #59 by 5:00 PM. You must show all work to receive credit.

1. (5 points each) Compute the following determinants:

$$\text{(a) } \det \begin{bmatrix} 1 & 2 & 1 & 3 \\ 2 & 0 & 5 & 0 \\ 0 & 1 & 3 & 7 \\ 0 & 5 & 0 & 5 \end{bmatrix} \quad \text{(b) } \det \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 3 & 2 & -3 & 3 \\ 0 & 2 & 0 & 2 & 0 \\ 1 & 5 & 3 & 5 & 4 \\ 1 & -2 & 2 & 6 & 5 \end{bmatrix}$$

2. (10 points) We say that a matrix is *upper triangular* if all entries below the *diagonal* (entries a_{ii} for $i = 1, \ldots, n$) are zero, and that it is *lower triangular* if all entries above the diagonal are zero. Use expansion by minors to show that the determinant of a matrix that is either upper or lower triangular is the product of the diagonal entries. In other words, show that

$$\det \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ 0 & a_{22} & \cdots & a_{2n} \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & a_{nn} \end{bmatrix} = \det \begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} = a_{11}a_{22}\cdots a_{nn}.$$

3. (10 points) Check that $y_1(x) = x + 1$, $y_2(x) = 1 + x^2$, and $y_3(x) = x^2 - x$ are solutions of the equation $y''' = 0$ for all x. Is $\{y_1, y_2, y_3\}$ a fundamental set?

4. (10 points) Check that $y_1(x) = 1$, $y_2(x) = e^{2x}$, and $y_3(x) = e^{-2x}$ are solutions of the equation $y''' - 4y' = 0$ for all x. Is $\{y_1, y_2, y_3\}$ a fundamental set?

5. (5 points each) From Section 9.2

   (a) #1 (b) #3 (c) #5 (d) #7 (e) #31 (f) #33

6. (10 points) From Section 9.2, #23.

7. (10 points each)

   (a) Find the general solution of $y'' - y' - 2y = 4 - x$, by first finding the general solution of the related homogeneous equation, then guessing a particular solution of the form $y_p(x) = Ax + B$ and solving for A and B.

   (b) Find the general solution of $y''' - y'' - 2y' = 4$, by first finding the general solution of the related homogeneous equation, then guessing a particular solution of the form $y_p(x) = Ax$ and solving for A.