MATH 2260 (Ordinary Differential Equations I) — Fall 2014
Homework #4

Due Date: Thursday, October 16, in class or in marking box #59 by 5:00 PM. You must show all work to receive credit.

1. (5 points each) From Section 4.1

   (a) #1

   (b) #3

   (c) #13

2. (5 points each) From Section 4.2

   (a) #5

   (b) #7

   (c) #11

3. (20 points) A further modification of the logistic growth equation includes a minimum threshold, T, required for the population to be viable. Consider the case where $0 < T < K$, where K is the carrying capacity defined in class. The resulting model is given as

$$\frac{dy}{dt} = -r\left(1 - \frac{y}{T}\right)\left(1 - \frac{y}{K}\right)y.$$

   (a) Is this equation autonomous?

   (b) For what values of y is $\frac{dy}{dt} > 0$? For what values of y is $\frac{dy}{dt} < 0$? Consider only values $y > 0$.

   (c) Using a partial fractions decomposition, find an impicit solution to this equation. For full credit, your answer should not include any logarithms (natural or otherwise!).

4. (10 points each) From Section 5.1

   (a) #1

   (b) #3

5. (10 points) Show that x^3 and $|x|^3$ are linearly independent for $-\infty < x < \infty$, but that they are not for $-\infty < x < 0$.

6. (10 points) Check that $y_1(x) = \sin(2x)$ and $y_2(x) = \sin(x)\cos(x)$ are solutions of $y'' + 4y = 0$ for $-\infty < x < \infty$. Is $\{y_1, y_2\}$ a fundamental set of solutions?

7. (10 points) Check that $y_1(x) = x^{-1}$ and $y_2(x) = x^{-2}$ are solutions of $x^2 y'' + 4xy' + 2y = 0$ for $0 < x < \infty$. Is $\{y_1, y_2\}$ a fundamental set of solutions?