MATH 2260 (Ordinary Differential Equations I) — Fall 2014 Homework #1

Due Date: Thursday, September 18, in class or in marking box #59 by 5:00 PM. You must show all work to receive credit.

- 1. (6 points) State the order of the following ODEs:
 - (a) $y'' + x^3 y = \sin(x)$

(b)
$$\sqrt{y^{(4)}} + (y'')^5 = e^{-x^3/2}$$

- (c) $\frac{d^3y}{dx^3} + \frac{d^5y}{dx^5} + \frac{dy}{dx} = y^9$
- 2. (12 points) For each of the following first-order ODEs, state if they are linear. If they are linear, state if they are homogeneous.
 - (a) $x^2y' + x^5y = 0$
 - (b) $x^3y' = x^7y^2$
 - (c) $\sin(x)y' + y = e^x$
 - (d) $y' + x\sin(y) = e^x$
- 3. (20 points) For each of the following ODEs, check if the given function is a solution to the ODE
 - (a) $xy' + y = 0, y(x) = \frac{1}{x}$
 - (b) $\sin(x)y' + \cos(x)y = 1, \ y(x) = \cos(x)$

(c)
$$y^{(4)} + 16y = 0, y(x) = e^{2x}$$

- (d) $y'' + 2y' + y = 0, y(x) = cxe^{-x}$
- 4. (12 points) Find the general solutions to the following equations
 - (a) $y' = x^2 e^{-x}$ (b) $y^{(3)} = \cos(x)$
- 5. (10 points) Find the solution to the initial value problem $y'' = e^x + e^{-x}$, y(0) = 1, y'(0) = 2
- 6. (10 points) Sketch the direction field and some integral curves for $y' = \frac{-y}{1+y^2}$.
- 7. (10 points) Use separation of variables to find the general solution to $\cos^2(x)y' + y = 0$.
- 8. (20 points) Use variation of parameters to find the general solution to $y' \frac{x}{x^2+1}y = e^{-x}(x^2+1)^{1/2}$.