1. (15 points) Consider the point $P(1,2,1)$ and the plane $x - y + 2z = 4$.
   (a) Find the distance from the point to the plane.
   (b) Find the point in the plane that is closest to P.

2. (15 points) For each set of vectors below, prove if they are linearly independent or dependent.
   (a) $\vec{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\vec{u}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\vec{u}_3 = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$, and $\vec{u}_4 = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$.
   (b) $\vec{u}_1 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$, $\vec{u}_2 = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$, and $\vec{u}_3 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$.

3. (15 points) Compute the products of the matrices below, if possible. If not possible, explain why.
   (a) AB for $A = \begin{bmatrix} 1 & -1 & 3 \\ -2 & 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 1 \\ -3 & 2 \end{bmatrix}$.
   (b) $B^T A$ for $A = \begin{bmatrix} 2 & 0 & -1 \\ 3 & 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 2 \end{bmatrix}$.
   (c) A^3 for $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$.

4. (40 points) Find all solutions, if any, to each of the given systems of equations. Express your answers in vector form.
   (a) $$\begin{aligned} x + y &= 3 \\ 2x + y &= 4 \end{aligned}$$
   (b) $$\begin{aligned} x - 2y + 2z &= 3 \\ 2x - 4y &= 2 \end{aligned}$$
   (c) $$\begin{aligned} x + y + z &= -2 \\ 2x + 2y - 2z &= -4 \\ -2x + y + 3z &= -5 \end{aligned}$$
   (d) $$\begin{aligned} x - z &= 1 \\ x + 2y + 2z &= 2 \\ 3x + 2y &= 3 \end{aligned}$$

5. (15 points) Let $\vec{u}$ and $\vec{v}$ be nonzero vectors in $\mathbb{R}^n$.
   (a) Give the formula for $\text{proj}_{\vec{v}}\vec{u}$.
   (b) Show that $\vec{v}$ is orthogonal to $\vec{u} - \text{proj}_{\vec{v}}\vec{u}$.