MATH 2050 **Practice Midterm 2** **Fall 2016**

Multiple choice: No partial credit will be given. 4 points each.

1. The projection of $\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 3 \end{bmatrix}$ onto $\vec{v} = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ is given by

   (a) $\begin{bmatrix} 3 \\ 3 \\ 1 \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1/2 \\ 1 \\ 0 \\ 3/2 \end{bmatrix}$ (d) $\frac{6}{7}\begin{bmatrix} 3 \\ 3 \\ 1 \\ 0 \end{bmatrix}$

2. Which elementary row operation transforms the linear system

$$\begin{aligned} 2x + \ \ y &= 2 \\ x + 2y &= 1 \end{aligned} \quad \text{into} \quad \begin{aligned} x - \ \ y &= 1 \\ x + 2y &= 1 \end{aligned} \ ?$$

   (a) $R2 \leftrightarrow R1$ (b) $R2 \leftarrow R2 - R1$ (c) $R1 \leftarrow R1 - R2$ (d) $R1 \leftarrow (1/2)R1$

3. Which variable is a free variable of the system $\begin{bmatrix} 1 & 0 & 2 & -3 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 0 & -4 \end{bmatrix}$?

   (a) x_1 (b) x_2 (c) x_3 (d) x_4

4. Which of the following is the soution of $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$?

   (a) $x = 2$, $y = -1$ (b) $x = -2$, $y = 1$ (c) $x = 0$, $y = 0$ (d) $x = 0$, $y = 3$

5. Given $\vec{u} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$, for which $\vec{w}$ is the set $\{\vec{u}, \vec{v}, \vec{w}\}$ linearly independent?

   (a) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ (c) $\begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

6. (15 points) Consider the plane $\pi : 2x - y + z = 0$.

   (a) Find two orthogonal vectors in the plane, π.

   (b) Find the projection of $\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ onto the plane, π.

7. (15 points) Consider $\vec{u}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\vec{u}_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$, $\vec{u}_3 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$, and $\vec{u}_4 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$.

   (a) Show that $\vec{u}_1$, $\vec{u}_2$, and $\vec{u}_3$ are linearly independent.

   (b) Are $\vec{u}_1$, $\vec{u}_2$, $\vec{u}_3$, and $\vec{u}_4$ linearly independent?

8. (10 points) Given $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, compute AB and BA.

9. (40 points) Find all solutions, if any, to each of the given systems of equations. Express your answers in vector form.

   (a) $\begin{aligned} 2x - y &= 3 \\ 4x - 2y &= 4 \end{aligned}$

   (b) $\begin{aligned} x - y + 2z &= -2 \\ 2y - z &= -4 \\ x + y + 3z &= -10 \end{aligned}$

   (c) $\begin{aligned} x + y + 3z &= 2 \\ -2x - 2y - 6z &= -4 \end{aligned}$

   (d) $\begin{aligned} x + y - 2z &= 3 \\ 2x - y + 3z &= 0 \\ x - 2y + 2z &= -3 \end{aligned}$