MATH 2050 **Practice Midterm 1** **Fall 2016**

Multiple choice: No partial credit will be given. 4 points each.

1. Which of the following vectors is parallel to $\begin{bmatrix} 0 \\ -2 \\ 3 \end{bmatrix}$?

   (a) $\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ (c) $\begin{bmatrix} 0 \\ -3 \\ -2 \end{bmatrix}$ (d) $\begin{bmatrix} 0 \\ 6 \\ -9 \end{bmatrix}$

2. Which of the following vectors is a unit vector?

   (a) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 3/5 \\ 0 \\ -4/5 \end{bmatrix}$ (c) $\begin{bmatrix} 2/5 \\ 2/5 \\ 1/5 \end{bmatrix}$ (d) $\begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$

3. What is the angle between vectors $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ -1 \end{bmatrix}$?

   (a) 0 radians (b) $\pi/6$ radians (c) $\pi/2$ radians (d) $3\pi/4$ radians

4. What is the cross product of vectors $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$?

   (a) $\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ (c) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ (d) $\begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$

5. What is the equation of the plane that passes through point $P(3, -2, 4)$ with normal vector $\begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$?

   (a) $2y - z = -4$ (b) $3x - 2y + 4z = -7$ (c) $2y - x = -7$ (d) $2y - z = -7$

6. (20 points) Consider the vectors $\vec{u} = \begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix}$, and $\vec{v} = \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}$.

   (a) Is $\vec{w} = \begin{bmatrix} 2 \\ 6 \\ -4 \end{bmatrix}$ a linear combination of $\vec{u}$ and $\vec{v}$?

   (b) Find a vector orthogonal to both $\vec{u}$ and $\vec{v}$

   (c) Find the area of the parallelogram with sides $\vec{u}$ and $\vec{v}$.

7. (20 points) Find the point of intersection of the lines $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} + t \begin{bmatrix} 5 \\ 0 \\ -3 \end{bmatrix}$ and $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + s \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$. Find the equation of the plane that includes both lines.

8. (10 points) Consider the points $A(0,1,2)$, $B(2,3,1)$, and $C(2,2,4)$.

   (a) Find the cosine of the angle between vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

   (b) Are $\overrightarrow{AB}$ and $\overrightarrow{AC}$ orthogonal?

9. (15 points) Consider the vectors $\vec{u} = \begin{bmatrix} 0 \\ 3 \\ -2 \end{bmatrix}$, and $\vec{v} = \begin{bmatrix} 1 \\ x \\ 3x^2 \end{bmatrix}$.

   (a) Find all values of x such that $\vec{u}$ and $\vec{v}$ are orthogonal.

   (b) Explain why there are no values of x such that $\vec{u}$ and $\vec{v}$ are parallel.

   (c) Give a non-zero vector $\vec{w}$ that is orthogonal to $\vec{v}$ for all values of x.

10. (15 points) Verify the *scalar triple product* identity, that, for any vectors $\vec{u}$, $\vec{v}$, and $\vec{z}$ in $\mathbb{R}^3$,

$$\vec{u} \cdot (\vec{v} \times \vec{z}) = \vec{v} \cdot (\vec{z} \times \vec{u}).$$