2016 AARMS-CRM-NSF Workshop on the Numerical Analysis of Singularly Perturbed Differential Equations

July 25-29, Saint Mary's University, Halifax, Nova Scotia, Canada

PROGRAM

AARMS (Atlantic Association for Research in the Mathematical Sciences), the AARMS Collaborative Research Group in Numerical Analysis and Scientific Computing, **Memorial University of Newfoundland**, and the Department of Mathematics and Computing Science, **Saint Mary's University**, will bring researchers interested in the numerical analysis of singularly perturbed differential equations and its applications to a 5-day workshop in beautiful Halifax, NS, Canada.

The meeting will consist of a 2-day short course on the numerical analysis of singularly perturbed differential equations taught by leading expert **Niall Madden (National University of Ireland, Galway)**, research level talks by experts in the numerical analysis of singularly perturbed differential equations, and talks by researchers with applied problems who have an interest in exploring the application of the numerical analysis of singularly perturbed differential equations to their problems. The final part of the workshop will be interactive, linking the applied researchers with numerical analysis experts to investigate the process of applying the numerical analysis of singularly perturbed differential equations to their problems.

VENUE

Sobey Building, **Saint Mary's University** (Room SB 260)

PLENARY SPEAKERS

Niall Madden

National University of Ireland, Galway

Natalia Kopteva

University of Limerick

Torsten Linß

University of Hagen

Martin Stynes

Beijing Computational Science Research Center

ORGANIZERS

Hermann Brunner, Ronald Haynes, Scott MacLachlan

Memorial University

Paul Muir

Saint Mary's University

David Iron, Theodore Kolokolnikov

Dalhousie University

APPLIED PROBLEM PRESENTERS

Adriana Dawes

Ohio State University

Alan Lindsay

University of Notre Dame

Andrew Bernoff

Harvey Mudd College

For more information: http://www.math.mun.ca/~smaclachlan/anasc_spde/

Dear Workshop Participants,

We welcome you to the 2016 AARMS/CRM/NSF Workshop on the numerical analysis of singularly perturbed differential equations, to be held July 25-29 at Saint Mary's University, Halifax, Nova Scotia, Canada. This workshop is the third in a series of workshops sponsored by the AARMS Collaborative Research Group in Numerical Analysis and Scientific Computing.

This year's workshop has four related components: a short course on the numerical analysis of singularly perturbed differential equations taught by Niall Madden (National University of Ireland, Galway), with an associated hands-on software session, plenary research talks on the latest advances in the numerical analysis of singularly perturbed differential equations by Niall Madden, Natalia Kopteva (University of Limerick), Torsten Linß (University of Hagen), and Martin Stynes (Beijing Computational Science Research Center), and applied problem presentations by Adriana Dawes (Ohio State University), Alan Lindsay (University of Notre Dame), and Andrew Bernoff (Harvey Mudd College), and interactive break-out sessions.

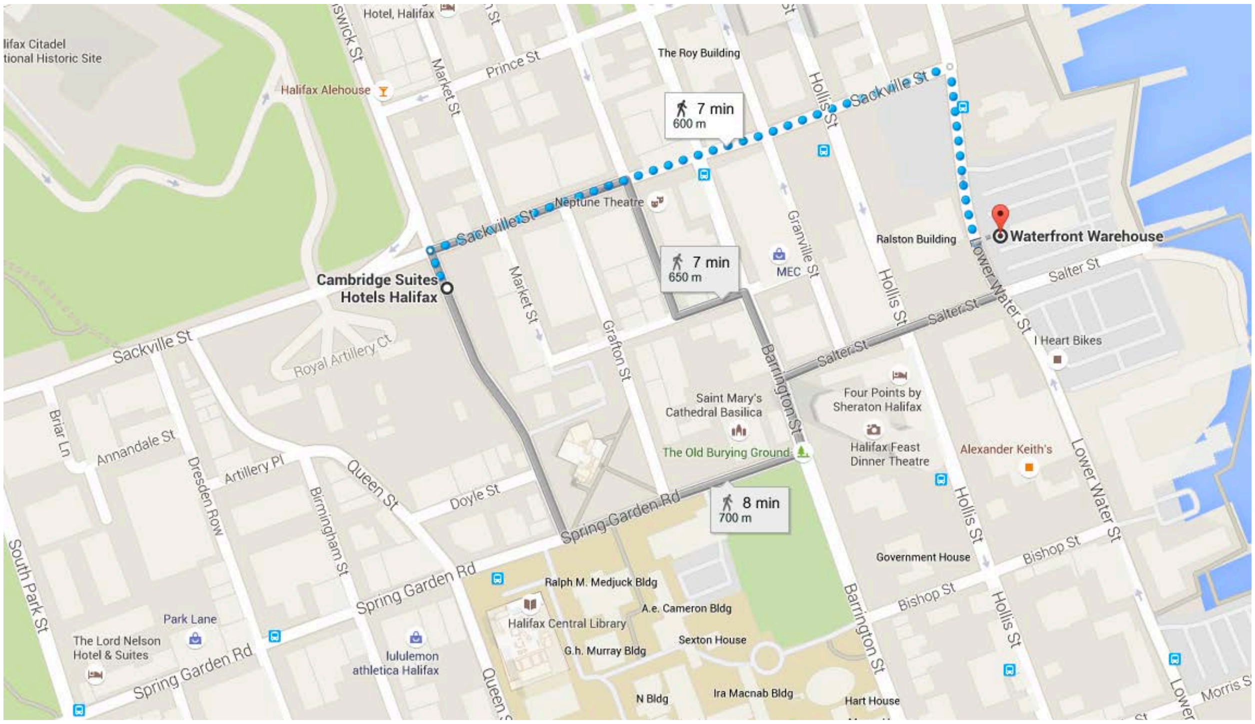
Our sincere thanks to our financial sponsors; these include the Atlantic Association for Research in the Mathematical Sciences, the Centre de Recherches Mathématiques, the National Science Foundation (USA), Memorial University of Newfoundland, and Saint Mary’s University.

This program contains the schedule for the workshop, as well as organizational information and a list of titles/abstracts for the speakers at the workshop.

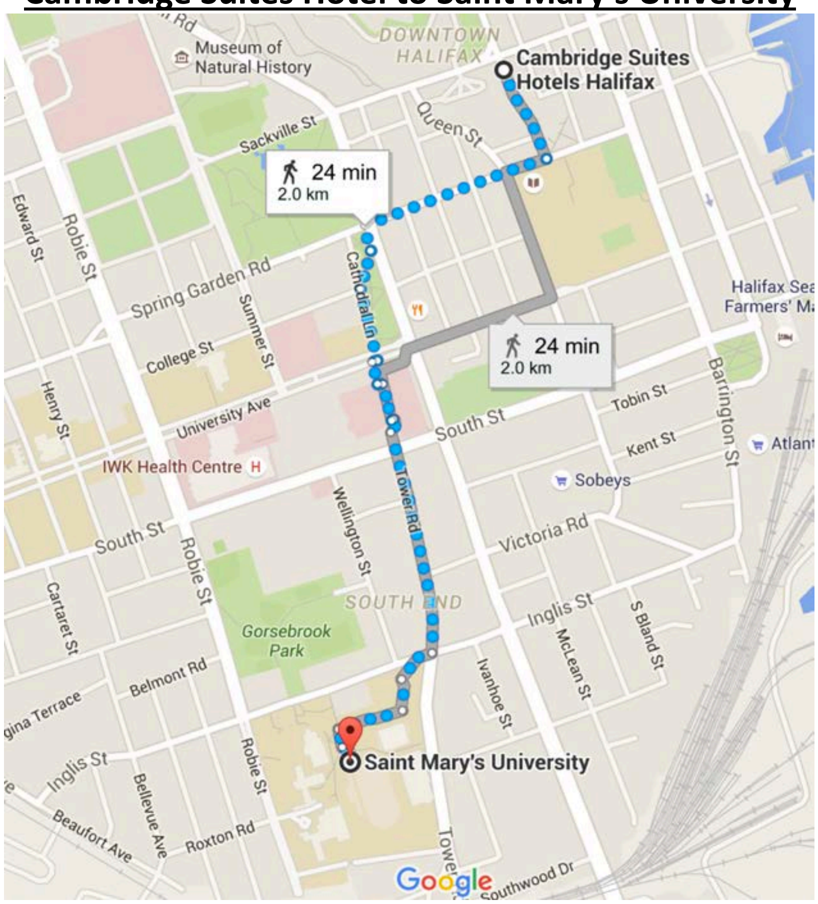
We wish you an interesting and productive time at the workshop and during your stay in Halifax!

Sincerely,
Hermann Brunner, Ronald Haynes, David Iron, Theodore Kolokolnikov, Scott MacLachlan, and Paul Muir (Co-organizers)

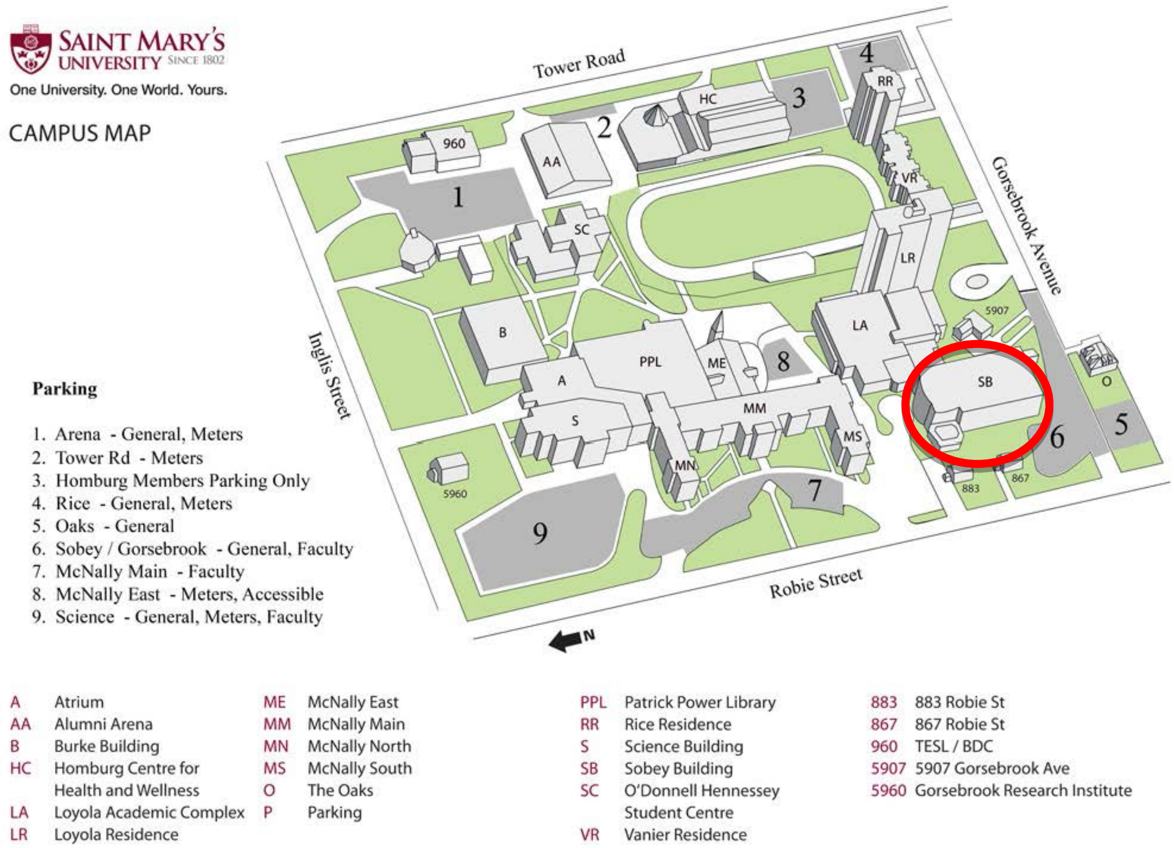
Cambridge Suites Hotel to Tug's Pub at the Waterfront Warehouse (Pub Night, July 28th)



Cambridge Suites Hotel to Saint Mary's University



Saint Mary's University Campus – Enter Sobey Building from Robie Street
Workshop will be held in SB 260



Parking

Parking passes can be purchased during the first morning (Monday, July 25th). Instructions on how to do so will be provided on Monday morning as the workshop begins.

WiFi

eduroam

Eduroam (**edu**cation **roam**ing) is the secure, world-wide roaming access service developed for the international research and education community.

Saint Mary's University is one of the Eduroam affiliated institutions; Eduroam is available on campus.

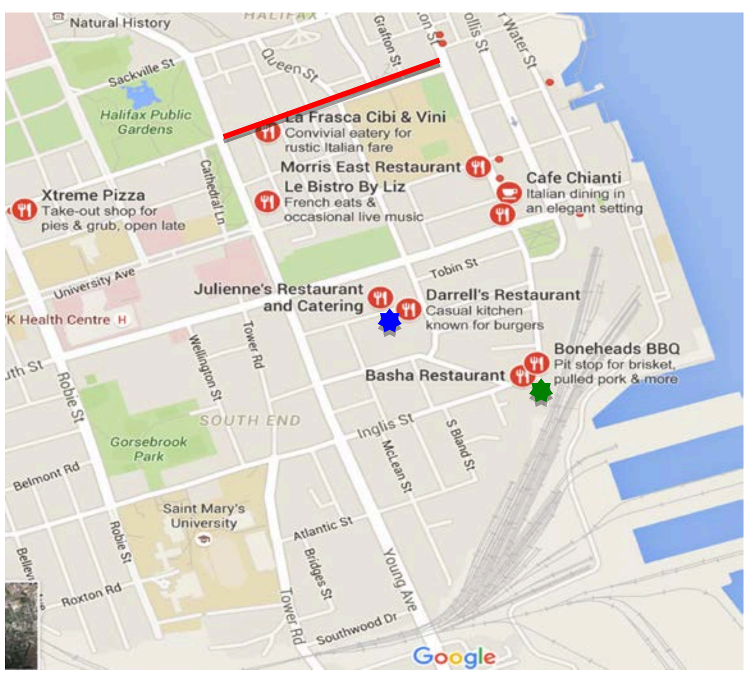
Alternatively, information will be provided at the beginning of the workshop on how to login using a guest account through the Saint Mary's University wireless network.

Food

- **On campus:**

- The Dockside, Tim Horton’s and the Cstore are in the **Loyola Academic Complex (LA)***
- The Atrium Coffee shop is in the **Atrium (A)***
- The Gorsebrook Lounge is in the **O’Donnell Hennesey Student Center (SC)***
- The Faculty Lounge is in **McNally East (ME)***
  *** See map on previous page**

- **Off campus:** There are many food establishments within a 15-20 minute walk from Saint Mary’s University. The map below gives some examples: Spring Garden Road **(Red)**, Fenwick Street **(Blue)**, Inglis Street **(Green)**.



Travel Reimbursement

For workshop participants who have been authorized to receive travel reimbursement, the instructions for completing the reimbursement process are given below. Note that the process for U.S.-based participants is different from the process for non-U.S. participants. Please save all original receipts associated with your travel claim, as well as original (paper) boarding passes for all flights to be reimbursed. Note that you will only be reimbursed up to the amount that you have been authorized to claim in advance by the conference organizer, Scott MacLachlan.

Non-U.S. Participants

Please get a copy of the travel claim form from Scott MacLachlan, and confirm your details with him. Original receipts are required for everything except meals. Airfares will need to show itinerary, as well as costs, and boarding passes are required. Taxi receipts need to have the name of the taxi company (a credit/debit card slip without this is not sufficient), and they need to indicate where from/to. Per Diem amounts are $50/day for meals plus $5/day for incidentals.

If any costs are in currencies other than Canadian, you will need to go to the Bank of Canada website:

http://www.bankofcanada.ca/rates/exchange/10-year-converter/

to convert to Canadian dollars based on the day the expense was incurred - this will need to be attached to the claim.

Travel claim forms require a complete home (university) address, an indication of whether you are faculty, a postdoc, or a graduate or undergraduate student, and must be signed as "claimant". Completed forms and receipts/documentation should be mailed to:

Ros English, Department of Mathematics and Statistics
Memorial University of Newfoundland, St. John's, NL Canada A1C 5S7

Make sure to include your e-mail address with the materials you send us in the event we need to follow up with you.

U.S.-based Participants

Travel claims for U.S.-based Participants will be processed through Tufts University. Please get a copy of the travel claim form from:

https://tufts.my.salesforce.com/sfc/p/#50000000BJax/a/50000000HAo2/SUugMCDMBlC8qsQWu7lWhWlCcweeLe.Fj6ZveFHu54I ,

and confirm your details with Scott MacLachlan. Original receipts are required for everything except meals. If you traveled by air, Tufts can only reimburse flights on a U.S. airline, or operated by a U.S. carrier - it MUST be a U.S. flight number. You must submit an itinerary that shows both the price you paid for the tickets and the method of payment, as well as all boarding passes (originals only, no copies!). It is important for you to keep all original documentation for ground transportation expenses, including an itemized receipt as well as a proof of payment. In order to be reimbursed for lodging, you must submit a receipt that indicates the cost of the hotel room has been paid. It must show the amount paid, the payment method, and that the balance is now zero. Only the cost of the room itself and tax will be reimbursed. Meal expenses will only be reimbursed through a per diem of $50 / day.

When expenses are not in US dollars, you must provide the official exchange rate for every day there is a foreign expense. Please use the website:

https://www.oanda.com/currency/converter/

to convert to US dollars, and include a printout of the exchange rate for each day there's an expense (using the "date" selector).

Completed forms and receipts/documentation should be mailed to:
Megan Monaghan, Tufts University, Department of Mathematics
503 Boston Ave, Medford, MA 02155 USA

Schedule

Short course and all talks will be in the Sobey Building, SB 260. Breakout sessions will be in SB260, SB 152, and SB 153.

	Mon, 25 July	Tues, 26 July	Weds, 27 July	Thurs, 28 July	Fri, 29 July
09:00	*Welcome/Coffee*				
09:15	Short Course	Short Course	Bernoff	Kopteva	Madden
10:00	*Coffee Break*				
10:15	Short Course	Short Course	Dawes Lindsay	Breakout	Breakout Wrapup
12:00	*Lunch*				
14:00	Short Course	Short Course	Breakout	Linß	
15:00	*Coffee Break*				
15:15	Short Course	Short Course	Breakout	Stynes	
16:00	Lab	Lab	Breakout	Breakout	
17:30	*Finish*				

Opening Reception

Cambridge Suites Hotel, 6:00pm-9:00pm, July 24, 2016 (See maps on Page 2)

Networking Event (Pub Night)

Tug's Pub, Waterfront Warehouse, 5:30pm-8:30pm, July 28, 2016 (See maps on Page 2)

Abstracts for Plenary Lecturers

Natalia Kopteva
Department of Mathematics and Statistics, University of Limerick, Ireland
natalia.kopteva@ul.ie

A posteriori error estimates on anisotropic meshes

Our goal is to obtain residual-type a posteriori error estimates on reasonably general anisotropic meshes for the semilinear reaction-diffusion equation

$$-\varepsilon^2 \triangle u + f(x, u) = 0, \tag{1}$$

as well as the convection-dominated convection-diffusion equation

$$-\varepsilon \triangle u + \bar{a} \cdot \nabla u + bu = f(x). \tag{2}$$

Both (1) and (2) are posed in a polygonal domain $\Omega \subset R^n$, n=2,3. Here $0 < \varepsilon \leq 1$. In (1), we also assume that f is continuous on $\Omega \times R$ and satisfies $f(\cdot; s) \in L_\infty(\Omega)$ for all $s \in R$, and the one-sided Lipschitz condition $f(x, y; v) - f(x, y; w) \geq C_f[v - w]$ whenever $v \geq w$, with some constant $C_f \geq 0$.

For (1), residual-type a posteriori error estimates in the maximum norm were recently given in [1] in the case of shape-regular triangulations. In [2], the consideration was restricted to Ω in R^2 and linear finite elements, but the focus shifted to more challenging anisotropic meshes, i.e. we allowed mesh elements to have extremely high aspect ratios. More recently, the analysis of [2] was extended to the error estimation in the energy norm [3]. To give a flavour of the results of [3], assuming that all mesh elements are anisotropic and almost non-obtuse, our first estimator reduces to

$$\varepsilon \|\nabla(u_h - u)\|_{L_2(\Omega)} + \|u_h - u\|_{L_2(\Omega)} \leq C \Big\{ \sum_{z \in \mathcal{N}} \min\{h_z H_z,\, \varepsilon H_z^2 h_z^{-1}\} \, \|\varepsilon J_z\|^2_{L_\infty(\gamma_z)} + \sum_{z \in \mathcal{N}} \|\min\{1,\, H_z \varepsilon^{-1}\}\, f_h^I\|^2_{L_2(\omega_z)} + \|f_h - f_h^I\|^2_{L_2(\Omega)} \Big\}^{1/2},$$

where C is independent of the diameters and the aspect ratios of elements in $\mathcal{T}$, and of ε. Here $f_h := f(\cdot, u_h)$, $\mathcal{N}$ is the set of nodes in $\mathcal{T}$, J_z is the standard jump in the normal derivative of the computed solution u_h across an element edge, ω_z is the patch of elements surrounding any $z \in \mathcal{N}$, γ_z is the set of edges in the interior of ω_z, $H_z = \operatorname{diam}(\omega_z)$, and $h_z \sim H_z^{-1}|\omega_z|$.

In this talk, we shall review [1, 2, 3], and then present more recent work in which the approach of [3] is extended to finite-element approximations of (2) on anisotropic meshes.

References

[1] A. Demlow and N. Kopteva, Maximum-norm a posteriori error estimates for singularly perturbed elliptic reaction-diffusion problems, Numer. Math., 2015, published electronically 14-Aug-2015.

[2] N. Kopteva, Maximum-norm a posteriori error estimates for singularly perturbed reaction-diffusion problems on anisotropic meshes, SIAM J. Numer. Anal., 53, 2015, pp. 2519–2544.

[3] N. Kopteva, Energy-norm a posteriori error estimates for singularly perturbed reaction-diffusion problems on anisotropic meshes, 2016, http://www.staff.ul.ie/natalia/pubs.html.

Torsten Linß
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A posteriori maximum-norm error estimates for parabolic equations

For classical and singularly perturbed parabolic equations, we present maximum norm a posteriori error estimates that, in the singulaly perturbed regime, hold uniformly in the small perturbation parameter. The parabolic equations are discretized in time using the backward Euler method, the Crank-Nicolson method and the discontinuous Galerkin dG(r) method. Both semidiscrete (no spatial discretation) and fully discrete cases will be considered. The analysis relies on Green's-function bounds and invokes elliptic reconsturcions and elliptic a posteriori error estimates.

Joint work with Natalia Kopteva (University of Limerick, Ireland)

Niall Madden and Stephen Russell
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Sparse grid methods for reaction-diffusion problems in two and three dimensions

We consider the numerical solution, by finite element methods, of singularly perturbed reaction-diffusion equations of the form

$$Lu := -\varepsilon^2 \Delta u + bu = f \quad \text{in } \Omega := (0,1)^d, \qquad u = 0 \text{ on } \partial\Omega,$$

for $d = 2, 3$. It is known that a standard Galerkin FEM with bilinear elements applied on a piecewise uniform mesh can solve the two-dimensional problems in a robust manner (see, e.g., Liu et al. [1]). We show how to extend this method and analysis to problems in three dimensions, using a solution decomposition due to Shishkin and Shishkina ([4, Chap. 3]).

This method is, however, not very scalable: the computational cost becomes prohibitively expensive for even moderate values of the discretisation parameter. To resolve this issue we consider the application of *sparse grid techniques*. These methods have many variants, including the hierarchical basis approach of Zenger [5], which is highly efficient, and the so-called two-scale method developed by Zhou and co-authors (e.g., [1]), which is less efficient, but much (arguably) much simpler in both theory and practice.

We first show how to unify these two approaches, resulting in a multiscale method [2, 3]. We then consider the generalisations of the two-scale technique for the three-dimensional problem.

References

[1] Fang Liu, Niall Madden, Martin Stynes, and Aihui Zhou. A two-scale sparse grid method for a singularly perturbed reaction–diffusion problem in two dimensions. *IMA J. Numer. Anal.*, 29(4):986–1007, 2009.

[2] Niall Madden and Stephen Russell. A multiscale sparse grid finite element method for a two-dimensional singularly perturbed reaction-diffusion problem. *Adv. Comput. Math.*, 41(6):987–1014, 2015.

[3] S. Russell and N. Madden. An introduction to the analysis and implementation of sparse grid finite element methods. *ArXiv e-prints*, November 2015.

[4] Grigory I. Shishkin and Lidia P. Shishkina. *Difference methods for singular perturbation problems*, volume 140 of *Chapman & Hall/CRC Monographs and Surveys in Pure and Applied Mathematics*. CRC Press, Boca Raton, FL, 2009.

[5] C. Zenger. Sparse grids. In *Parallel algorithms for partial differential equations (Kiel, 1990)*, volume 31 of *Notes Numer. Fluid Mech.*, pages 241–251. Vieweg, Braunschweig, 1991.

Martin Stynes
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Some open problems in the numerical solution of singularly perturbed differential equations

Some open questions in the numerical analysis of singularly perturbed differential equations are discussed. These include whether certain convergence results in various norms are optimal, when supercloseness is obtained in finite element solutions, and whether the analysis of the Il'in-Allen-Southwell scheme can be extended from 1D to 2D problems.

Abstracts for Problem Presenters

Andrew Bernoff
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Problems in diffusion and absorption: How fast can you hit a target with a random walk?

Consider the stamen of a flower waiting to catch a grain of pollen, a lymphocyte waiting to be stimulated by an antigen to produce antibodies, or an anteater randomly foraging for an ant nest to plunder. Each of these problems can be modeled as a diffusive process with a mix of reflecting and absorbing boundary conditions. One can characterize the agent (pollen, antigen, anteater) finding its target (stamen, lymphocyte, ant nest) as a first passage time (FPT) problem for the distribution of the time when a particle executing a random walk is absorbed. In this talk we will examine a hierarchy of FPT problems. I will show how to exploit dimensional analysis, asymptotics and exact solutions in formulating models that are amenable to analysis. In these problems the dimension of the domain plays a surprisingly strong role in the dynamics observed. Finally I will discuss some numerical strategies (particle methods, boundary integral representations) for attacking these problems and the benefits and pitfalls associated with them.

Parts of this work are collaborative with Alan Lindsay (University of Notre Dame).

Adriana Dawes
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How to make an organ: Specifying cell fate during development

Multicellular organisms are made of highly differentiated cells that perform very specific functions (think of hair versus heart cells). Yet all of these cells originated at some point in development from the same progenitor cell. In this talk, I will discuss new models of cell fate specification during development of the egg laying structure in nematode worms, a structure that is highly conserved across all nematode worms. A large biologically based model indicates that external signals may be responsible for observed species-specific responses to drug treatments. Further investigation using a simplified two variable model demonstrates that shifts in bifurcation planes between two nematode species can account for the difference in response. Taken together, these results suggest that variations in key parameters can allow for species-specific responses while preserving an identical outcome under normal conditions.

Alan Lindsay
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Singular perturbation problems in microscopic elastic-electrostatic interfaces

In this talk I will discuss the deformation of elastic beams and membranes by electric fields. These problems are fundamental interactions in the functioning of modern nanotechnology and microscopic self-assembly processes. These contact events are manifested as a singularities in the governing PDEs and require specialized analytical and numerical treatments to accurately characterize. In this talk I will discuss singular perturbation techniques and adaptive numerical methodologies for resolving solutions up to and beyond these singular events. The post contact dynamics are described by detailed interfacial dynamical laws and validated by adaptive moving mesh methods. We find new stable equilibrium states which raises the potential for bistability in the system. Estimates on the bistable parameter range are found by means of detailed singular perturbation analysis. This is joint work with Joceline Lega (Arizona), Karl Glasner (Arizona) and Kelsey Dipietro (Notre Dame).