MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

WORKSHEET **MATH 3202** SPRING 2019

For practice only. Not to be submitted.

Note: The following textbook problems are useful practice for the topics covered on this worksheet:

- Section 16.5, #s 1–8, 13–18, 23–31
- Section 16.8, #s 2–10, 13–15
- Section 16.9, #s 1–14

1. For each of the following, calculate the curl and divergence of $\mathbf{F}$. Use the curl to identify whether or not $\mathbf{F}$ is conservative.

   (a) $\mathbf{F}(x, y, z) = \langle z^2 - y^2,\ z^2 - x^2,\ y^2 - x^2 \rangle$

   (b) $\mathbf{F}(x, y, z) = \langle z\sin(y),\ xz\cos(y),\ x\sin(y) \rangle$

   (c) $\mathbf{F}(x, y, z) = \left\langle \dfrac{y}{xz},\ \dfrac{y\ln(x)}{z},\ -\dfrac{y^2\ln(x)}{2z^2} \right\rangle$

2. Prove that if $\mathbf{F}(x, y, z)$ is a vector field and $g(x, y, z)$ is a scalar function then

$$\operatorname{div}(g\mathbf{F}) - g\operatorname{div}(\mathbf{F}) = \mathbf{F} \cdot \nabla g.$$

3. Consider the vector field $\mathbf{F} = \langle xy,\ z - y,\ -2x \rangle$ and let the surface S be the portion of the plane $2x + 8y + z = 8$ which lies in the first octant. Verify Stokes' Theorem by computing both $\oint_{\partial S} \mathbf{F} \cdot d\mathbf{r}$ and $\iint_S \operatorname{curl}(\mathbf{F}) \cdot d\mathbf{S}$.

4. Use Stokes' Theorem to evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle e^{x^4},\ x^2 - \sin(y^3),\ 3x + \ln(z) \rangle$ and C is the rectangle with vertices $(0, 0, 4)$, $(3, 0, 4)$, $(3, 2, 4)$ and $(0, 2, 4)$.

5. Use Stokes' Theorem to evaluate $\iint_S \operatorname{curl}(\mathbf{F})\, d\mathbf{S}$ where $\mathbf{F} = \langle z,\ y,\ x \rangle$ and S is the upper hemisphere $(x - 2)^2 + y^2 + z^2 = 4$ (that is, the hemisphere for $z \geq 0$).

PLEASE TURN OVER

6. Consider the vector field $\mathbf{F} = \langle x,\ y,\ xyz \rangle$ and let E be the solid consisting of the portion of the paraboloid $z = x^2 + y^2$ and its interior lying below the plane $z = 1$. Verify the Divergence Theorem by computing both $\displaystyle\iint_{\partial E} \mathbf{F} \cdot d\mathbf{S}$ and $\displaystyle\iiint_{E} \operatorname{div}(\mathbf{F})\, dV$.

7. Use the Divergence Theorem to evaluate $\displaystyle\iint_{S} \mathbf{F} \cdot d\mathbf{S}$ where

$$\mathbf{F} = \left\langle y^8 \ln(\cos(z)),\ x^5 + y^3 z,\ e^{\sin(x)+\cos(y)} \right\rangle$$

and S is the union of the six faces of the rectangular prism defined by $[0,2] \times [0,3] \times [0,4]$.

8. Suppose S is a closed surface bounding a solid E and $\mathbf{F}$ and $\mathbf{G}$ are vector fields such that $\mathbf{F} = \operatorname{curl}(\mathbf{G})$. Determine $\displaystyle\iint_{S} \mathbf{F} \cdot d\mathbf{S}$.

9. Find the minimum value of the function $f(x, y) = x + 2y$ on the elliptic cylinder $3x^2 + 4y^2 = 3$.

10. Find the point on the plane $x + 2y + 3z = -14$ which is closest to the origin.

11. If the edges of an open-topped rectangular box have a total length of 35 cm, find the largest possible surface area.