# MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

| TEST 1 | **MATHEMATICS 3202** | JUNE 17TH, 2019 |
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| **Name** | **MUN Number** |
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1. Consider the vector function $\mathbf{r}(t) = \langle 2\cos(t),\ 5\sec^2(t)\rangle$.

[2] (a) Find $\lim_{t\to\frac{\pi}{4}} \mathbf{r}(t)$ or explain why the limit does not exist.

[2] (b) Determine $\mathbf{r}'(t)$.

[2] (c) Evaluate $\int \mathbf{r}(t)\, dt$.

[3] (d) Express the curve corresponding to $\mathbf{r}(t)$ as an equation in $(x, y)$-coordinates. Does $\mathbf{r}(t)$ trace out the entire curve with this equation, or only a portion of it?

2. Consider the vector function $\mathbf{r}(t) = \left\langle \frac{2}{3}t^{\frac{3}{2}},\ t,\ \frac{2\sqrt{2}}{3}t^{\frac{3}{2}} \right\rangle$ for $t \geq 0$.

[6] (a) Derive the arclength function $s(t)$.

[3] (b) Reparametrise $\mathbf{r}(t)$ in terms of its arclength.

[5] (c) Evaluate the line integral $\int_C \frac{3y+1}{3}\, ds$ where $C$ is the curve traced out by $\mathbf{r}(t)$ for $0 \leq t \leq 1$.

[12] 3. Consider the curve traced out by $\mathbf{r}(t) = \langle 3t,\ 4\sin(t),\ 4\cos(t)\rangle$.

(a) Find the unit tangent vector $\mathbf{T}(t)$.

(b) Compute the curvature $\kappa(t)$.

(c) Find the unit normal vector $\mathbf{N}(t)$.

(d) Verify that $\mathbf{T}(t)$ and $\mathbf{N}(t)$ are orthogonal.

(e) Find the binormal vector $\mathbf{B}(t)$.

[5] 4. Use the Chain Rule to prove the first Frenet-Serret formula

$$\frac{d\mathbf{T}}{ds} = \kappa\mathbf{N}$$

for any curve traced out by a vector function $\mathbf{r}(t)$, where $\mathbf{T}$ is the unit tangent vector, $\kappa$ is the curvature, and $\mathbf{N}$ is the unit normal vector.

[40]