MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 1	**MATH 3202**	SPRING 2019

SOLUTIONS

[3] 1. We must consider the domains of each of the component functions. The function $\arcsin(t)$ requires $-1 \le t \le 1$. The function $\ln(3t)$ requires $3t > 0$ so $t > 0$. The function $\dfrac{1}{4t-3}$ requires $t \ne \frac{3}{4}$. Thus the domain of $\mathbf{r}(t)$ is given by $\left\{t \,|\, 0 < t \le 1,\ t \ne \frac{3}{4}\right\}$.

[2] 2. We let $\mathbf{r}_0 = \langle 4, -2, 5\rangle$ and $\mathbf{v} = \langle 1, 0, -3\rangle$ so a direction vector is given by

$$\mathbf{v} - \mathbf{r}_0 = \langle -3, 2, -8\rangle.$$

Hence the line is the graph of the function

$$\mathbf{r}(t) = \mathbf{r}_0 + t(\mathbf{v} - \mathbf{r}_0) = \langle 4, -2, 5\rangle + t\langle -3, 2, -8\rangle = \langle 4 - 3t, -2 + 2t, 5 - 8t\rangle.$$

[2] 3. (a) From the second component of $\mathbf{r}(t)$, we see that we must have $t - 4 = -2$ so $t = 2$. Substituting this into the first component, $t^2 = 4$ as required. But substituting it into the third component, we have $2t^3 = 16 \ne 4$. Thus the point $(4, -2, 16)$ lies on the space curve, but the point P does not.

[2] (b) Again the second component of $\mathbf{r}(t)$ indicates that $t - 4 = -5$ so $t = -1$. The first component becomes $t^2 = 1$ and the third component becomes $2t^3 = -2$, as required. Hence the point Q does lie on the space curve.

[3] 4. (a) The parametric equations are

$$x = 1 + 4\sin(5t), \quad y = 4\cos(5t) - 7.$$

So observe that

$$(x-1)^2 = [4\sin(5t)]^2 = 16\sin^2(5t)$$

and likewise

$$(y+7)^2 = [4\cos(5t)]^2 = 16\cos^2(5t),$$

so

$$(x-1)^2 + (y+7)^2 = 16\sin^2(5t) + 16\cos^2(5t) = 16.$$

Hence the circle has radius $\sqrt{16} = 4$ and centre $(1, -7)$.

[1] (b) We can consider, say, $t = 0$ at which $\mathbf{r}(0) = \langle 1, -3\rangle$. If we allow t to slightly increase, we can see that $4\sin(5t)$ will increase and so x increases. On the other hand, $4\cos(5t)$ will decrease and so y decreases as well. Thus the circle is being traced out in a clockwise direction.

[1] (c) At $t = \frac{\pi}{5}$ we have $\mathbf{r}\left(\frac{\pi}{5}\right) = \langle 1, -11\rangle$. Thus we have moved from the top point of the circle at $t = 0$ to the bottom point at $t = \frac{\pi}{5}$, moving clockwise. Hence, under this restriction, the righthand semi-circle is traced out.

[3] 5. (a) The parametric equations are

$$x = 3\cos(t), \quad y = 4\cos(t).$$

Thus $\cos(t) = \frac{1}{3}x$ and so

$$y = 4\left(\frac{1}{3}x\right) = \frac{4}{3}x.$$

The corresponding graph is a <u>line</u>.

[3] (b) The parametric equations are

$$x = 3\cos(t), \quad y = 4\sin(t) = 4\sqrt{1-\cos^2(t)}.$$

Thus $\cos(t) = \frac{1}{3}x$ and so

$$y = 4\sqrt{1-\left(\frac{1}{3}x\right)^2} = 4\sqrt{1-\frac{x^2}{9}}.$$

To make it easier to classify the corresponding curve, we can rewrite this as

$$y^2 = 16\left(1-\frac{x^2}{9}\right) \quad \Longrightarrow \quad \frac{x^2}{9}+\frac{y^2}{16} = 1,$$

so the corresponding graph is an <u>ellipse</u>.

[3] (c) The parametric equations are

$$x = 3\cos(t), \quad y = 4\sin^2(t) = 4[1-\cos^2(t)].$$

Thus $\cos(t) = \frac{1}{3}x$ and so

$$y = 4\left[1-\left(\frac{1}{3}x\right)^2\right] = 4-\frac{4}{9}x^2.$$

Hence the corresponding graph is a <u>parabola</u>.

[3] (d) The parametric equations are

$$x = 3\sec(t), \quad y = 4\tan(t) = 4\sqrt{\sec^2(t)-1}.$$

Thus $\sec(t) = \frac{1}{3}x$ and so

$$y = 4\sqrt{\left(\frac{1}{3}x\right)^2-1} = 4\sqrt{\frac{x^2}{9}-1}.$$

We can rewrite this as

$$y^2 = 16\left(\frac{x^2}{9}-1\right) \quad \Longrightarrow \quad \frac{x^2}{9}-\frac{y^2}{16} = 1,$$

so the corresponding graph is a <u>hyperbola</u>.

[3] 6. (a) To allow for the possibility that Glaaki and Hastur may occupy the same space at different times, we will use s as the parameter for Hastur's trajectory, so

$$\mathbf{r}_H = \langle 4s - 6, 4s^2, -8s^{-1} \rangle.$$

We set $\mathbf{r}_G(t) = \mathbf{r}_H(s)$ so

$$t = 4s - 6, \quad t^2 = 4s^2, \quad t^3 = -8s^{-1}.$$

Substituting the first equation into the second, we obtain

$$\begin{aligned} (4s-6)^2 &= 4s^2 \\ 12s^2 - 48s + 36 &= 0 \\ 12(s-3)(s-1) &= 0, \end{aligned}$$

so $s = 3$ or $s = 1$.

When $s = 3$, the first equation indicates that $t = 6$. However, these values do not satisfy the third equation because $t^3 = 216$ but $-8s^{-1} = -\frac{8}{3}$.

When $s = 1$, we have $t = -2$. Now $t^3 = -8 = -8s^{-1}$. Hence Glaaki and Hastur occupy the same point — the point represented by the vector $\langle -2, 4, -8 \rangle$ — but at two different times. We conclude that they are intersecting.

[3] (b) Proceeding as before, we have the system of equations

$$t = 2s - 1, \quad t^2 = 2s^2 - s, \quad t^3 = \sqrt{s}.$$

Substituting the first equation into the second, we obtain

$$\begin{aligned} (2s-1)^2 &= 2s^2 - s \\ 2s^2 - 3s + 1 &= 0 \\ (2s-1)(s-1) &= 0 \end{aligned}$$

and so $s = \frac{1}{2}$ or $s = 1$.

When $s = \frac{1}{2}$, the first equation indicates that $t = 0$. However, these values do not satisfy the third equation because $t^3 = 0$ but $\sqrt{s} = \frac{\sqrt{2}}{2}$.

When $s = 1$, we have $t = 1$. Now $t^3 = 1 = \sqrt{s}$. Hence Glaaki and Hastur occupy the same point — the point represented by the vector $\langle 1, 1, 1 \rangle$ — at $t = 1$. We conclude that they are now colliding.

[3] 7. (a) We consider the limits of each component function:

$$\lim_{t \to 0} \frac{1}{e^t} = 1, \quad \lim_{t \to 0} \frac{6t^2}{t - t^2} = \lim_{t \to 0} \frac{6t}{1-t} = 0, \quad \lim_{t \to 0} \cos\left(\frac{\pi}{t+1}\right) = \cos(\pi) = -1.$$

Hence

$$\lim_{t \to 0} \mathbf{r}(t) = \langle 1, 0, -1 \rangle.$$

[2] (b) This time, observe that the limit of the second component function is

$$\lim_{t \to 1} \frac{6t^2}{t - t^2}$$

which produces a $\frac{6}{0}$ form. Hence this limit does not exist, and so regardless of the limits of the other component functions, we can immediately conclude that $\lim_{t\to 1} \mathbf{r}(t)$ does not exist.

[3] (c) The limits at infinity of the component functions are

$$\lim_{t \to \infty} \frac{1}{e^t} = 0, \quad \lim_{t \to \infty} \frac{6t^2}{t - t^2} = \lim_{t \to \infty} \frac{6}{\frac{1}{t} - 1} = -6, \quad \lim_{t \to \infty} \cos\left(\frac{\pi}{t+1}\right) = \cos(0) = 1.$$

Hence

$$\lim_{t \to \infty} \mathbf{r}(t) = \langle 0, -6, 1 \rangle.$$