MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 8	**Math 3202**	SPRING 2019

Due: Friday, August 2nd, 2019 at 1:00pm. SHOW ALL WORK.

> **Note:** The following textbook problems are useful practice for the topics covered on this assignment:
> - Section 16.3, #s 3–10, 12–18
> - Section 16.4, #s 1–14, 19

1. Determine whether each of the following vector fields is conservative. If so, find the potential function f such that $\mathbf{F} = \nabla f$.

   (a) $\mathbf{F}(x, y) = \langle 4xy^3 - 5,\ 6x^2y^2 \rangle$

   (b) $\mathbf{F}(x, y) = \langle x\ln(y) + 1,\ y\ln(x) + y^2 \rangle$

   (c) $\mathbf{F}(x, y) = \langle \sin(y) - \sin(x),\ x\cos(y) - \sin(y) \rangle$

   (d) $\mathbf{F}(x, y, z) = \langle xy^2z^3,\ x^2yz^3,\ x^2y^2z^2 \rangle$

   (e) $\mathbf{F}(x, y, z) = \langle xy^2z^2 + 2,\ x^2yz^2 + 3,\ x^2y^2z + 4 \rangle$

2. Use the Fundamental Theorem of Line Integrals to evaluate $\displaystyle\int_C \nabla f \cdot d\mathbf{r}$ where $f(x, y) = xy^2$ and C is the quarter-circle $x^2 + y^2 = 4$ from $(2, 0)$ to $(0, 2)$.

3. Use the Fundamental Theorem of Line Integrals to evaluate $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$ where

$$\mathbf{F} = \langle 2x - 3z,\ 3y^2,\ -3x \rangle$$

   is a conservative vector field and C is the indicated smooth curve.

   (a) C is the portion of the helix defined by $\mathbf{r}(t) = \langle 3\cos(t),\ 3\sin(t), 4t \rangle$ for $0 \leq t \leq \frac{\pi}{2}$

   (b) C is the line joining the point $(2, -1, -3)$ to the point $(8, 4, 4)$

   (c) C is the union of the curves C_1, C_2 and C_3, where C_1 is the plane curve consisting of the parabola $y = 2x^2$ from $(0, 0, 0)$ to $(1, 2, 0)$, C_2 is the line joining $(1, 2, 0)$ to $(1, 2, 1)$, and C_3 is the curve defined by $\mathbf{r}(t) = \langle e^t,\ 2e^t,\ e^{2t} \rangle$ for $0 \leq t \leq 1$

PLEASE TURN OVER

4. Use Green's Theorem to evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$ in each case.

   (a) $\mathbf{F} = \langle y\sqrt{x^2+9},\ e^y \rangle$ where C is the triangle with vertices $(0,0)$, $(4,0)$ and $(4,8)$

   (b) $\mathbf{F} = \langle -x^2y,\ xy^2 \rangle$ where C is the unit circle $x^2+y^2=1$

   (c) $\mathbf{F} = \langle x+y^2,\ x^2+3xy \rangle$ where $C = \partial D$, the boundary of the region D which lies between the curves $y = x^2$ and $y = \sqrt{x}$

5. **[BONUS]** The folium of Descartes (depicted in Figure 1) is one of many curves which is not the graph of a function, but is instead defined implicitly, in this case by the equation

$$x^3 + y^3 = 3xy.$$

   As such, it is not possible to use a double integral to find the area of the region inside the loop of the graph.

   (a) Show that the folium of Descartes can be parametrised by the function

$$\mathbf{r}(t) = \left\langle \frac{3t}{1+t^3},\ \frac{3t^2}{1+t^3} \right\rangle.$$

   (b) Use Green's Theorem to find the area of the loop in the folium of Descartes. (Note that the full graph is traced by $-\infty < t < \infty$, $t \neq -1$, but the loop itself is traced out by $0 \le t < \infty$.)

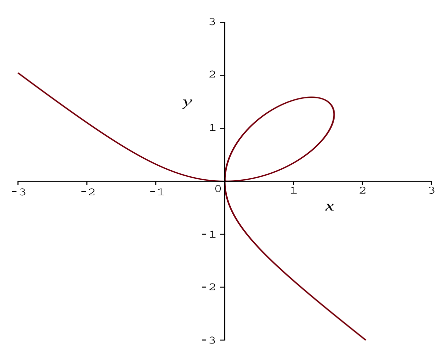

Figure 1: The folium of Descartes.