MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 5 | **Math 3202** | SPRING 2019

Due: Friday, July 5th, 2019 at 1:00pm. SHOW ALL WORK.

> **Note:** The following textbook problems are useful practice for the topics covered on this assignment:
> - Section 14.6, #s 7-17, 19–34, 37, 41–46, 49–61

1. Find the directional derivative of the given function in the direction of the vector $\mathbf{v}$ at the point P.

   (a) $f(x, y) = x^2y^3, \quad \mathbf{v} = \langle -3,\ 4\rangle, \quad P(2, -1)$

   (b) $f(x, y, z) = xe^{-yz}, \quad \mathbf{v} = \langle 1,\ -1,\ -2\rangle, \quad P(1, 0, -3)$

2. A drone is exploring the thermal currents above an active volcano. It has been determined that the temperature can be modelled by the function

$$T(x, y, z) = \frac{\sin(x)\cos(y)}{z^2 + 1}$$

   where temperature is measured in degrees Celsius and distance is measured in metres. The drone is currently situated at the origin.

   (a) Find the maximum rate of change of the temperature at the origin, and the direction in which it occurs.

   (b) The drone begins moving towards the point $P(10, 5, 10)$. Find the rate of change of the temperature at the origin in this direction.

3. Consider the function

$$f(x, y) = x^2 + \sin(xy).$$

   Identify any unit vectors which point in a direction for which $f(x, y)$ does not instantaneously change at the point $P(1, 0)$.

4. Prove that if $f(x, y)$ and $g(x, y)$ are differentiable functions then

$$\nabla(fg) = f\nabla g + g\nabla f.$$

PLEASE TURN OVER

5. Consider the ellipsoid

$$5x^2 + y^2 + 3z^2 = 1$$

at the point $P(1, 4, -1)$.

(a) Find an equation of the tangent plane to the ellipsoid at P.

(b) Find an equation of the normal line to the ellipsoid at P.

6. Find an equation of any plane tangent to the hyperbolic paraboloid

$$z = \frac{x^2}{4} - \frac{y^2}{3}$$

which is parallel to the plane $x + 4y + 6z = 0$.

7. Show that every normal line to the sphere $(x-a)^2 + (y-b)^2 + (z-c)^2 = R^2$ passes through the centre (a, b, c) of the sphere.