MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

Assignment 1	**Math 3202**	Spring 2019

Due: Friday, May 24th, 2019 by 1:00pm. SHOW ALL WORK.

> **Note:** The following textbook problems are useful practice for the topics covered on this assignment:
> - Section 13.1, #s 1–14, 17–20, 41, 49, 50

1. Find the domain of the function $\mathbf{r}(t) = \left\langle \arcsin(t), \ln(3t), \dfrac{1}{4t-3} \right\rangle$.

2. Find a vector parametrisation of the line which passes through the points $P(4, -2, 5)$ and $Q(1, 0, -3)$.

3. Consider the space curve defined by $\mathbf{r}(t) = \langle t^2, t-4, 2t^3 \rangle$. Determine whether each of the following points lies on the curve.

   (a) $P(4, -2, 4)$

   (b) $Q(1, -5, -2)$

4. The graph of $\mathbf{r}(t) = \langle 1 + 4\sin(5t), 4\cos(5t) - 7 \rangle$ is a circle.

   (a) Find the radius and the centre of the circle.

   (b) Determine whether the orientation of the curve (that is, the direction traced out by the curve as t increases) is clockwise or counterclockwise.

   (c) Identify the part of the curve traced out if the domain is restricted to $0 \le t \le \frac{\pi}{5}$.

5. For each of the following vector functions, express the corresponding curve as an equation in (x, y)-coordinates. If possible, classify the graph as a line, parabola, circle, ellipse or hyperbola.

   (a) $\mathbf{r}(t) = \langle 3\cos(t), 4\cos(t) \rangle$

   (b) $\mathbf{r}(t) = \langle 3\cos(t), 4\sin(t) \rangle$

   (c) $\mathbf{r}(t) = \langle 3\cos(t), 4\sin^2(t) \rangle$

   (d) $\mathbf{r}(t) = \langle 3\sec(t), 4\tan(t) \rangle$

6. Scientists are observing an asteroid named Glaaki whose trajectory can be modelled by the vector function

$$\mathbf{r}_G(t) = \langle t, t^2, t^3 \rangle.$$

The scientists are looking for asteroids that may interact with Glaaki. If two asteroids will occupy the same point in space at the same time, they are classified as *colliding.* If they will occupy the same point in space at different times, they are classified as *intersecting.* Otherwise, they are *non-interacting.*

(a) The scientists identify a second asteroid of interest, named Hastur. Initially they model its trajectory as

$$\mathbf{r}_H(t) = \langle 4t - 6, 4t^2, -8t^{-1} \rangle.$$

Determine whether Glaaki and Hastur are colliding, intersecting or non-interacting.

(b) Later, new information helps the scientists to more accurately represent Hastur's trajectory as

$$\mathbf{r}_H(t) = \langle 2t - 1, 2t^2 - t, \sqrt{t} \rangle.$$

Now determine whether Glaaki and Hastur are colliding, intersecting or non-interacting.

7. Given

$$\mathbf{r}(t) = \left\langle \frac{1}{e^t}, \frac{6t^2}{t - t^2}, \cos\left(\frac{\pi}{t+1}\right) \right\rangle,$$

evaluate each limit or explain why it does not exist.

(a) $\lim_{t \to 0} \mathbf{r}(t)$

(b) $\lim_{t \to 1} \mathbf{r}(t)$

(c) $\lim_{t \to \infty} \mathbf{r}(t)$