9. FIRST WE COMPUTE $\int_{\partial S} \underline{F} \cdot d\underline{r}$ WHERE

∂S IS THE UNIT CIRCLE $x^2+y^2=1$ WHICH IS PARAMETRISED BY $\underline{r}(t) = \langle \cos(t), \sin(t), 0 \rangle$ FOR $0 \leq t \leq 2\pi$. THEN

$$\underline{F}(\underline{r}(t)) = \langle 2\cos(t)\sin(t), \cos(t), \sin(t) \rangle$$

$$\underline{r}'(t) = \langle -\sin(t), \cos(t), 0 \rangle$$

$$\underline{F}(\underline{r}(t)) \cdot \underline{r}'(t) = -2\cos(t)\sin^2(t) + \cos^2(t)$$

THEN

$$\int_{\partial S} \underline{F} \cdot d\underline{r} = \int_0^{2\pi} \left[-2\cos(t)\sin^2(t) + \cos^2(t)\right] dt$$

$$= \int_0^{2\pi} \cos^2(t)\,dt - 2\int_0^{2\pi} \cos(t)\sin^2(t)\,dt$$

$$= \frac{1}{2}\int_0^{2\pi} [1+\cos(2t)]\,dt - 2\int_0^{2\pi} \cos(t)\sin^2(t)\,dt$$

$$= \frac{1}{2}\left[t + \frac{1}{2}\sin(2t)\right]_0^{2\pi} - 2\int_0^{2\pi} \cos(t)\sin^2(t)\,dt$$

LET $u = \sin(t)$

$du = \cos(t)\,dt$

$t=0 \to u=0$

$t=2\pi \to u=0$

$$= \pi - 2\int_0^{2\pi} \cos(t)\sin^2(t)\,dt$$

$$= \pi - 2\int_0^0 u^2\,du$$

$$= \pi - 2 \cdot 0$$

$$\boxed{= \pi}$$

NEXT WE COMPUTE $\iint_S \text{curl}(\underline{F}) \cdot d\underline{S}$

$$\text{curl}(\underline{F}) = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy & x & y+z \end{vmatrix}$$

$$= \underline{i}(1-0) - \underline{j}(0-0) + \underline{k}(1-2x)$$

$$= \langle 1, 0, 1-2x \rangle$$

A NORMAL TO S IS $\underline{n} = \langle 2x, 2y, 1 \rangle$ SO

$$\text{curl}(\underline{F}) \cdot \underline{n} = 2x + 0 + 1 - 2x = 1$$

THUS $\iint_S \text{curl}(\underline{F}) \cdot dS = \iint_S 1 \, dA = \iint_S dA$

IN POLAR COORDINATES THE REGION OF INTEGRATION IS DEFINED BY

$$0 \le r \le 1$$
$$0 \le \theta \le 2\pi$$

SO $\iint_S \text{curl}(\underline{F}) \cdot dS = \int_0^{2\pi} \int_0^1 r \, dr \, d\theta$

$$= \int_0^{2\pi} \left[\frac{1}{2} r^2 \right]_{r=0}^{r=1} d\theta$$

$$= \frac{1}{2} \int_0^{2\pi} d\theta$$

$$= \frac{1}{2} \left[\theta \right]_0^{2\pi}$$

$$= \frac{1}{2} \cdot 2\pi$$

$$\boxed{= \pi}$$