8. $\underline{F} = \langle P(x,y,z), Q(x,y,z), R(x,y,z) \rangle$

$$\operatorname{curl}(\underline{F}) = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$$

$$= \underline{i}\left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right) - \underline{j}\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) + \underline{k}\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)$$

$$= \langle R_y - Q_z, P_z - R_x, Q_x - P_y \rangle$$

$$\operatorname{div}(\operatorname{curl}(\underline{F})) = \frac{\partial}{\partial x}(R_y - Q_z) + \frac{\partial}{\partial y}(P_z - R_x) + \frac{\partial}{\partial z}(Q_x - P_y)$$

$$= R_{yx} - Q_{zx} + P_{zy} - R_{xy} + Q_{xz} - P_{yz}$$

$\therefore$ BY CLAIRAULT'S THM, $R_{yx} = R_{xy}$, $Q_{xz} = Q_{zx}$, $P_{zy} = P_{yz}$ SO

$$\operatorname{div}(\operatorname{curl}(\underline{F})) = R_{yx} - R_{yx} + Q_{xz} - Q_{xz} + P_{zy} - P_{zy}$$

$$= 0$$