6. IS $\underline{F}(x,y,z)$ CONSERVATIVE?

$P(x,y,z) = x^2$ $\quad Q(x,y,z) = \cos(y)\sin(z)$ $\quad R(x,y,z) = \sin(y)\cos(z)$

$P_y = 0$ $\quad Q_x = 0$ $\quad R_x = 0$

$P_z = 0$ $\quad Q_z = \cos(y)\cos(z)$ $\quad R_y = \cos(y)\cos(z)$

SINCE $P_y = Q_x$, $P_z = R_x$, AND $Q_z = R_y$ THEN $\underline{F}(x,y,z)$ $\underline{\text{IS}}$ CONSERVATIVE SO THERE EXISTS A FUNCTION $f(x,y,z)$ SUCH THAT $\underline{F}(x,y,z) = \nabla f(x,y,z)$.

THEN
$$f(x,y,z) = \int x^2\,dx$$
$$= \frac{1}{3}x^3 + C(y,z)$$
$$f_y(x,y,z) = C_y(y,z) = \cos(y)\sin(z)$$
$$C(y,z) = \int \cos(y)\sin(z)\,dy$$
$$= \sin(y)\sin(z) + C(z)$$
$$f(x,y,z) = \frac{1}{3}x^3 + \sin(y)\sin(z) + C(z)$$
$$f_z(x,y,z) = \sin(y)\cos(z) + C'(z) = \sin(y)\cos(z)$$
$$C'(z) = 0$$
$$C(z) = C$$

SO $f(x,y,z) = \frac{1}{3}x^3 + \sin(y)\sin(z) + C$

BY THE FUNDAMENTAL THM OF LINE INTEGRALS,
$$\int_C \underline{F}\cdot d\underline{r} = f(\underline{r}(2)) - f(\underline{r}(0))$$
$$= f(5, e^2, e^4) - f(1,1,1)$$
$$= \frac{125}{3} + \sin(e^2)\sin(e^4) - \left[\frac{1}{3} + \sin(1)\sin(1)\right]$$
$$= \frac{124}{3} + \sin(e^2)\sin(e^4) - \sin^2(1)$$