5. IN SPHERICAL COORDINATES, THE SPHERES BECOME

$$\rho = 2 \quad \text{AND} \quad \rho = 1$$

THUS E IS DEFINED BY

$$1 \le \rho \le 2$$
$$0 \le \varphi \le \pi/2$$
$$0 \le \theta \le 2\pi$$

THE INTEGRAND BECOMES

$$z(x^2+y^2+z^2) = \rho\cos(\varphi)\left[\rho^2\sin^2(\varphi)\cos^2(\theta) + \rho^2\sin^2(\varphi)\sin^2(\theta) + \rho^2\cos^2(\varphi)\right]$$
$$= \rho\cos(\varphi)\left[\rho^2\sin^2(\varphi) + \rho^2\cos^2(\varphi)\right]$$
$$= \rho\cos(\varphi)\cdot\rho^2$$
$$= \rho^3\cos(\varphi)$$

ALSO, $dV = \rho^2\sin(\varphi)\,d\rho\,d\varphi\,d\theta$ SO

$$\iiint_E z(x^2+y^2+z^2)\,dV = \int_0^{2\pi}\int_0^{\pi/2}\int_1^2 \rho^3\cos(\varphi)\cdot\rho^2\sin(\varphi)\,d\rho\,d\varphi\,d\theta$$
$$= \int_0^{2\pi}\int_0^{\pi/2}\int_1^2 \rho^5\cos(\varphi)\sin(\varphi)\,d\rho\,d\varphi\,d\theta$$
$$= \int_0^{2\pi}\int_0^{\pi/2}\left[\frac{1}{6}\rho^6\cos(\varphi)\sin(\varphi)\right]_{\rho=1}^{\rho=2} d\varphi\,d\theta$$
$$= \frac{21}{2}\int_0^{2\pi}\int_0^{\pi/2}\cos(\varphi)\sin(\varphi)\,d\varphi\,d\theta$$

LET $u = \sin(\varphi)$ $\qquad \varphi = 0 \to u = 0$

$du = \cos(\varphi)\,d\varphi$ $\qquad \varphi = \pi/2 \to u = 1$

THE INTEGRAL BECOMES

$$\frac{21}{2}\int_0^{2\pi}\int_0^1 u\,du\,d\theta$$

$$= \frac{21}{2}\int_0^{2\pi}\left[\frac{1}{2}u^2\right]_{u=0}^{u=1} d\theta$$

$$= \frac{21}{4}\int_0^{2\pi} d\theta$$

$$= \frac{21}{4}\left[\theta\right]_0^{2\pi}$$

$$= \frac{21}{4}\cdot 2\pi$$

$$\boxed{= \frac{21\pi}{2}}$$