MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

WORKSHEET | **MATH 2260** | SPRING 2019

For practice only. Not to be submitted.

1. Consider a function $f(t)$ with Laplace transform $\mathcal{L}\{f(t)\} = F(s)$. Prove that if $k > 0$ is a constant then
$$\mathcal{L}\{f(kt)\} = \frac{1}{k}F\left(\frac{s}{k}\right).$$

2. Use known Laplace transforms and the fact that
$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$
to find the Laplace transform of $\cosh(kt)$, where k is a constant.

3. For each of the following functions $F(s)$, determine a function $f(t)$ such that $\mathcal{L}\{f(t)\} = F(s)$.

   (a) $F(s) = \dfrac{1}{2s - 6}$

   (b) $F(s) = \dfrac{4s - 1}{s^2 + 16}$

   (c) $F(s) = \dfrac{s - 3}{s^2 - 2s + 5}$

   (d) $F(s) = \dfrac{2}{s} - \dfrac{1}{s + 2}$

4. Use the Laplace transform to solve each of the following initial value problems.

   (a) $\dfrac{dy}{dt} + 3y = 2e^{-t}, \quad y(0) = 1$

   (b) $\dfrac{dy}{dt} - 2y = e^{2t}\cos(3t), \quad y(0) = 0$

   (c) $\dfrac{d^2y}{dt^2} - 5\dfrac{dy}{dt} + 4y = 0, \quad y(0) = -2, \quad y'(0) = 7$

   (d) $\dfrac{d^2y}{dt^2} + 2\dfrac{dy}{dt} + 10y = 0, \quad y(0) = 0, \quad y'(0) = 3$

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5. Use the Laplace transform to solve each of the following systems of differential equations.

   (a) The system

$$\frac{dx}{dt} = 3x - 2y$$
$$\frac{dy}{dt} = 4x + 7y$$

   where $x(0) = 3$ and $y(0) = 2$.

   (b) The system

$$\frac{dx}{dt} = x - 4y + e^t$$
$$\frac{dy}{dt} = x + y$$

   where $x(0) = -1$, $y(0) = 0$.

6. Find the Laplace transform of each of the following functions.

   (a) $f(t) = u_2(t)e^t$

   (b) $f(t) = \begin{cases} \cos(3t-12)+4, & t \geq 4 \\ 4, & t < 4 \end{cases}$

   (c) $f(t) = \begin{cases} t, & 1 \leq t < 3 \\ 0, & t < 1 \quad \text{or} \quad t \geq 3 \end{cases}$

7. For each of the following functions $F(s)$, determine a function $f(t)$ such that $\mathcal{L}\{f(t)\} = F(s)$.

   (a) $F(s) = \dfrac{e^{-2s}}{s-7}$

   (b) $F(s) = \dfrac{e^{-3s}s}{s^2 - 4s + 29}$

   (c) $F(s) = \dfrac{e^{-2s} - e^{-5s}}{s^2 - 2s + 1}$

8. Solve the initial value problem

$$\frac{d^2y}{dt^2} + 9y = u_\pi(t)\cos(t-\pi), \quad y(0) = 0, \quad y'(0) = 2.$$