MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

TEST 2 **MATHEMATICS 2260** JULY 19TH, 2019

Name	**MUN Number**

1. Consider the initial value problem given by the equation

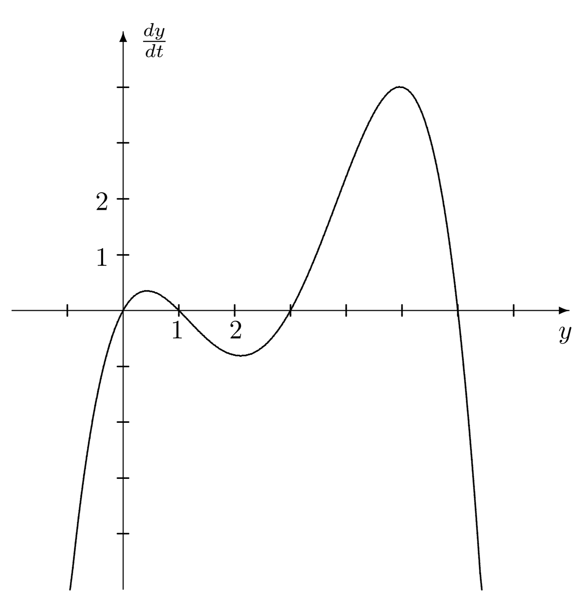
$$\cos(t)\frac{dy}{dt} + \frac{y}{t} = \frac{\cos(t)}{2t-1}$$

and the indicated initial condition. Give the interval of definition in each case. Do **NOT** find the solution.

[3] (a) $y(1) = 0$

[3] (b) $y(-1) = \pi$

[8] 2. The graph of $\frac{dy}{dt}$ vs. y for some autonomous equation $\frac{dy}{dt} = f(y)$ is given below. Identify the fixed points of the ODE, and classify them according to their stability. What is the long-term behaviour of the solution (that is, $\lim_{t\to\infty} y$) if $y(0) = 2$? What is the long-term behaviour of the solution if $y(0) = 4$?



3. Solve each of the following equations.

[4] (a) $9\frac{d^2y}{dt^2} - 4y = 0$

[4] (b) $\frac{d^2y}{dt^2} - 8\frac{dy}{dt} + 16y = 0$

[4] 4. Determine a second-order linear homogeneous equation with constant coefficients for which $y = 3\cos(7t)$ is a solution.

5. Consider the equation

$$t^2 \frac{d^2y}{dt^2} - 6y = 0.$$

One solution of this equation is $y_1 = t^3$.

[10] (a) Use the method of **reduction of order** to find a distinct second solution y_2 to the equation.

[4] (b) Use the Wronskian to prove that y_1 and y_2 form a fundamental set of solutions.

[40]