MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

Assignment 2	**MATH 2260**	Spring 2019

SOLUTIONS

[2] 1. (a) 3rd order, linear

[2] (b) 3rd order, nonlinear

[2] (c) 2nd order, nonlinear

[2] (d) 4th order, linear

[2] 2. (a) Note that

$$\frac{dy}{dt} = 1 \quad \text{and} \quad \frac{d^2y}{dt^2} = 0,$$

so

$$t^2\frac{d^2y}{dt^2} - t\frac{dy}{dt} + y = t^2(0) - t(1) + t = 0.$$

Hence $y = t$ is a solution of the given equation.

[2] (b) Note that

$$\frac{dy}{dt} = \frac{1}{t} \quad \text{and} \quad \frac{d^2y}{dt^2} = -\frac{1}{t^2},$$

so

$$t^2\frac{d^2y}{dt^2} - t\frac{dy}{dt} + y = t^2\left(-\frac{1}{t^2}\right) - t\left(\frac{1}{t}\right) + \ln(t) = -2 + \ln(t) \neq 0$$

for all t. Hence $y = \ln(t)$ is not a solution of the given equation.

[2] (c) Note that

$$\frac{dy}{dt} = \ln(t) + 1 \quad \text{and} \quad \frac{d^2y}{dt^2} = \frac{1}{t},$$

so

$$t^2\frac{d^2y}{dt^2} - t\frac{dy}{dt} + y = t^2\left(\frac{1}{t}\right) - t[\ln(t) + 1] + t\ln(t) = t - t\ln(t) - t + t\ln(t) = 0.$$

Hence $y = t\ln(t)$ is also a solution of the given equation.

[4] 3. (a) Observe that

$$p(t) = -\frac{4}{t} \quad \text{and} \quad g(t) = 6t.$$

Hence the integrating factor is

$$\mu = e^{-4\int \frac{dt}{t}} = e^{-4\ln(t)} = e^{\ln(t^{-4})} = t^{-4}.$$

Multiplying both sides of the given equation by μ, we obtain

$$\begin{aligned} t^{-4}\frac{dy}{dt} - 4t^{-5}y &= 6t^{-3} \\ \frac{d}{dt}[t^{-4}y] &= 6t^{-3} \\ t^{-4}y &= 6\int t^{-3}\,dt \\ &= -3t^{-2} + C \\ y &= -3t^2 + Ct^4. \end{aligned}$$

[4] (b) Since

$$p(t) = 5 \quad \text{and} \quad g(t) = e^{-2t},$$

the integrating factor is

$$\mu = e^{5\int dt} = e^{5t}.$$

Multiplying both sides of the given equation by μ results in

$$\begin{aligned} e^{5t}\frac{dy}{dt} + 5e^{5t}y &= e^{5t}e^{-2t} \\ \frac{d}{dt}[e^{5t}y] &= e^{3t} \\ e^{5t}y &= \int e^{3t}\,dt \\ &= \frac{1}{3}e^{3t} + C \\ y &= \frac{1}{3}e^{-2t} + Ce^{-5t}. \end{aligned}$$

[4] (c) First we rewrite the equation as

$$\frac{dy}{dt} + \cot(t)y = 1,$$

so clearly

$$p(t) = \cot(t) \quad \text{and} \quad g(t) = 1.$$

The integrating factor is

$$\mu = e^{\int \cot(t)\,dt} = e^{\ln(\sin(t))} = \sin(t).$$

Multiplying both sides of the given equation by μ yields

$$\begin{aligned}\sin(t)\frac{dy}{dt} + \cos(t)y &= \sin(t)\\ \frac{d}{dt}[\sin(t)y] &= \sin(t)\\ \sin(t)y &= \int \sin(t)\,dt\\ &= -\cos(t) + C\\ y &= -\cot(t) + C\csc(t).\end{aligned}$$

[4] (d) Again, we must begin by rewriting the equation:

$$\frac{dy}{dt} + 5t^4 y = t^4.$$

Now we can see that

$$p(t) = 5t^4 \quad \text{and} \quad g(t) = t^4,$$

and so the integrating factor is

$$\mu = e^{5\int t^4\,dt} = e^{t^5}.$$

Multiplying both sides of the given equation by μ gives us

$$\begin{aligned}e^{t^5}\frac{dy}{dt} + 5t^4 e^{t^5} y &= t^4 e^{t^5}\\ \frac{d}{dt}[e^{t^5} y] &= t^4 e^{t^5}\\ e^{t^5} y &= \int t^4 e^{t^5}\,dt\\ &= \frac{1}{5}e^{t^5} + C\\ y &= \frac{1}{5} + Ce^{-t^5}.\end{aligned}$$

Note that the last integral can be evaluated using u-substitution with $u = t^5$.

[4] (e) We rewrite the given equation as

$$\frac{dy}{dt} + \frac{t}{t-1}y = \frac{1}{t-1}$$

so

$$p(t) = \frac{t}{t-1} \quad \text{and} \quad g(t) = \frac{1}{t-1}.$$

The integrating factor is

$$\mu = e^{\int \frac{t}{t-1}\,dt} = e^{\int \frac{t-1+1}{t-1}\,dt} = e^{\int \left(1+\frac{1}{t-1}\right)dt} = e^{t+\ln(t-1)} = e^t e^{\ln(t-1)} = (t-1)e^t.$$

Multiplying both sides of the given equation by μ brings us to

$$\begin{aligned}(t-1)e^t\frac{dy}{dt}+te^ty&=e^t\\ \frac{d}{dt}[(t-1)e^ty]&=e^t\\ (t-1)e^ty&=\int e^t\,dt\\ &=e^t+C\\ y&=\frac{1}{t-1}+\frac{C}{(t-1)e^t}.\end{aligned}$$

[2] 4. (a) Substituting the initial condition into the general solution gives

$$\begin{aligned}0&=-3(2^2)+C(2^4)\\ 0&=-12+16C\\ C&=\frac{3}{4}.\end{aligned}$$

Hence the particular solution is

$$y=-3t^2+\frac{3}{4}t^4.$$

[2] (b) Substituting the initial condition into the general solution gives

$$\begin{aligned}\frac{7}{3}&=\frac{1}{3}e^0+Ce^0\\ \frac{7}{3}&=\frac{1}{3}+C\\ 2&=C.\end{aligned}$$

Hence the particular solution is

$$y=\frac{1}{3}e^{-2t}+2e^{-5t}.$$

[2] (c) Substituting the initial condition into the general solution gives

$$\begin{aligned}-1&=-\cot\left(\frac{\pi}{4}\right)+C\csc\left(\frac{\pi}{4}\right)\\ -1&=-1+C\sqrt{2}\\ C&=0.\end{aligned}$$

Hence the particular solution is simply

$$y=-\cot(t).$$