MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 1 | **MATH 2260** | SPRING 2019

SOLUTIONS

[5] 1. (a) We use integration by parts, letting $w = t$ so $dw = dt$, and $dv = \cos^2(t)\,dt$ so

$$v = \int \cos^2(t)\,dt = \int \frac{1+\cos(2t)}{2}\,dt = \frac{1}{2}t + \frac{1}{4}\sin(2t).$$

Thus

$$\begin{aligned}\int t\cos^2(t)\,dt &= t\left[\frac{1}{2}t + \frac{1}{4}\sin(2t)\right] - \int \left[\frac{1}{2}t + \frac{1}{4}\sin(2t)\right]\,dt \\ &= \frac{1}{2}t^2 + \frac{1}{4}t\sin(2t) - \frac{1}{4}t^2 + \frac{1}{8}\cos(2t) + C \\ &= \frac{1}{4}t^2 + \frac{1}{4}t\sin(2t) + \frac{1}{8}\cos(2t) + C.\end{aligned}$$

[5] (b) Again we use integration by parts, letting $w = x$ so $dw = dx$, and $dv = \sec^2(x)\,dx$ so $v = \tan(x)$. Thus

$$\begin{aligned}\int x\sec^2(x)\,dx &= x\tan(x) - \int \tan(x)\,dx \\ &= x\tan(x) + \ln|\cos(x)| + C.\end{aligned}$$

[5] (c) We decompose the integrand into partial fractions:

$$\frac{4y+3}{y^3+y^2-8y-12} = \frac{4y+3}{(y-3)(y+2)^2} = \frac{A}{y-3} + \frac{B}{y+2} + \frac{D}{(y+2)^2}.$$

Cross-multiplication gives

$$4y+3 = A(y+2)^2 + B(y-3)(y+2) + D(y-3)$$

so $A = \frac{3}{5}$, $B = -\frac{3}{5}$, $D = 1$. Thus

$$\begin{aligned}\int \frac{4y+3}{y^3+y^2-8y-12}\,dy &= \int \left[\frac{\frac{3}{5}}{y-3} - \frac{\frac{3}{5}}{y+2} + \frac{1}{(y+2)^2}\right] \\ &= \frac{3}{5}\ln|y-3| - \frac{3}{5}\ln|y+2| - \frac{1}{y+2} + C.\end{aligned}$$

[5] (d) Observe that we can simplify the integrand:

$$\int t^2 e^{\frac{1}{2}\ln(t^3+4)}\,dt = \int t^2 e^{\ln\left(\sqrt{t^3+4}\right)}\,dt = \int t^2\sqrt{t^3+4}\,dt.$$

Let $u = t^3+4$ so $du = 3t^2\,dt$ and $\frac{1}{3}\,du = t^2\,dt$. The integral becomes

$$\begin{aligned}\int t^2 e^{\frac{1}{2}\ln(t^3+4)}\,dt &= \frac{1}{3}\int \sqrt{u}\,du \\ &= \frac{1}{3}\cdot\frac{2}{3}u^{\frac{3}{2}} + C \\ &= \frac{2}{9}(t^3+4)^{\frac{3}{2}} + C.\end{aligned}$$

[5] (e) We complete the square:

$$\begin{aligned}7+12x-4x^2 &= -4\left[x^2-3x-\frac{7}{4}\right] \\ &= -4\left[\left(x^2-3x+\frac{9}{4}\right)-4\right] \\ &= 16-4\left(x-\frac{3}{2}\right)^2 \\ &= 16-(2x-3)^2.\end{aligned}$$

Thus

$$\int \frac{1}{\sqrt{7+12x-4x^2}}\,dx = \int \frac{1}{\sqrt{16-(2x-3)^2}}\,dx.$$

Let $u = 2x-3$ so $du = 2\,dx$ and $\frac{1}{2}\,du = dx$. We have

$$\begin{aligned}\int \frac{1}{\sqrt{7+12x-4x^2}}\,dx &= \frac{1}{2}\int \frac{1}{\sqrt{16-u^2}}\,du \\ &= \frac{1}{2}\arcsin\left(\frac{u}{4}\right) + C \\ &= \frac{1}{2}\arcsin\left(\frac{2x-3}{4}\right) + C.\end{aligned}$$

[5] (f) We can rewrite the integrand as

$$\int \frac{e^{3y}}{\csc(y)}\,dy = \int e^{3y}\sin(y)\,dy$$

and use integration by parts with $w = e^{3y}$ so $dw = 3e^{3y}\,dy$, and $dv = \sin(y)\,dy$ so $v = -\cos(y)$. (Note that we could also let $w = \sin(y)$ and $dv = e^{3y}\,dy$.) Then

$$\int e^{3y}\sin(y)\,dy = -e^{3y}\cos(y) + 3\int e^{3y}\cos(y)\,dy.$$

Now we use integration by parts a second time, this time setting $dv = \cos(y)\,dy$ so $v = \sin(y)$. We obtain

$$\begin{aligned}\int e^{3y}\sin(y)\,dy &= -e^{3y}\cos(y) + 3\left[e^{3y}\sin(y) - 3\int e^{3y}\sin(y)\right]\\ &= -e^{3y}\cos(y) + 3e^{3y}\sin(y) - 9\int e^{3y}\sin(y)\,dy\\ 10\int e^{3y}\sin(y)\,dy &= -e^{3y}\cos(y) + 3e^{3y}\sin(y) + C\\ \int e^{3y}\sin(y)\,dy &= -\frac{1}{10}e^{3y}\cos(y) + \frac{3}{10}e^{3y}\sin(y) + C.\end{aligned}$$

[5] (g) We decompose the integrand into partial fractions:

$$\frac{t+9}{t^3+9t} = \frac{t+9}{t(t^2+9)} = \frac{A}{t} + \frac{Bt+D}{t^2+9}.$$

Cross-multiplication gives

$$t + 9 = A(t^2+9) + (Bt+D)t$$

so $A = D = 1$, $B = -1$. Thus

$$\begin{aligned}\int \frac{t+9}{t^3+9t}\,dt &= \int \left(\frac{1}{t} + \frac{-t+1}{t^2+9}\right)dt\\ &= \int \left(\frac{1}{t} - \frac{t}{t^2+9} + \frac{1}{t^2+9}\right)dt\\ &= \ln|t| - \frac{1}{2}\ln(t^2+9) + \frac{1}{3}\arctan\left(\frac{t}{3}\right) + C.\end{aligned}$$

Note that the middle term can be integrated via u-substitution with $u = t^2 + 9$. The absolute value has been suppressed because $t^2 + 9$ is always positive.

[5] (h) Let $u = \sqrt{y}$ so $du = \frac{1}{2\sqrt{y}}\,dy$ and $2u\,du = dy$. The integral becomes

$$\int \sqrt{y}e^{\sqrt{y}}\,dy = 2\int u^2e^u\,du.$$

Now we use integration by parts with $w = u^2$ so $dw = 2u\,du$, and $dv = e^u\,du$ so $v = e^u$. We obtain

$$\begin{aligned}\int \sqrt{y}e^{\sqrt{y}}\,dy &= 2\left[u^2e^u - 2\int ue^u\,du\right]\\ &= 2u^2e^u - 4\int ue^u\,du.\end{aligned}$$

We use integration by parts again, this time setting $w = u$ so $dw = du$. Thus

$$\begin{aligned}\int \sqrt{y}e^{\sqrt{y}}\,dy &= 2u^2e^u - 4\left[ue^u - \int e^u\,du\right] \\ &= 2u^2e^u - 4ue^u + 4e^u + C \\ &= 2ye^{\sqrt{y}} - 4\sqrt{y}e^{\sqrt{y}} + 4e^{\sqrt{y}} + C.\end{aligned}$$