MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 7 **MATH 2260** SPRING 2019

Due: Wednesday, July 31st, 2019 at 1:00pm. SHOW ALL WORK.

1. Use the method of variation of parameters to find the general solution of each of the following equations. (Note that each of the corresponding homogeneous equations was solved in Question 1 on Assignment 5; you do not have to derive those results again.)

   (a) $\dfrac{d^2y}{dt^2} + 8\dfrac{dy}{dt} + 16y = t^{-2}e^{-4t}$

   (b) $9\dfrac{d^2y}{dt^2} + y = \sec\left(\dfrac{t}{3}\right)$

2. Given that y_1 and y_2 are solutions of the corresponding homogeneous equation, use the method of variation of parameters to solve the nonhomogeneous equation.

   (a) $t^2\dfrac{d^2y}{dt^2} - 5t\dfrac{dy}{dt} + 8y = \sqrt{t}, \quad y_1 = t^4, \quad y_2 = t^2$

   (b) $t^2\dfrac{d^2y}{dt^2} + 3t\dfrac{dy}{dt} + y = t\ln(t), \quad y_1 = t^{-1}, \quad y_2 = t^{-1}\ln(t)$

3. A mass m is attached to a large spring with spring constant 5 g/sec^2, causing it to stretch from its equilibrium position by 39.2 m.

   (a) Determine the value of m (in grams).

   (b) Find the value of the damping coefficient γ (in grams per second) which would result in the spring being critically damped.

   (c) Suppose that the mass (measured in grams) is driven by a decaying exponential force of the form $g(t) = 5e^{-t}$. Assuming that the damping coefficient is given by the value of γ (in grams per second) described in part (b), indicate the second-order equation which models this situation, and solve it using an appropriate non-homogeneous technique.

   (d) Now suppose that the mass (measured in grams) is driven instead by a decaying exponential force of the form $g(t) = 5e^{-\frac{1}{2}t}$. Again assuming that the damping coefficient is given by the value of γ (in grams per second) described in part (b), indicate the second-order equation which models this situation, and solve it using an appropriate non-homogeneous technique.