

7. a) THE CHARACTERISTIC EQN IS

$$r^2 + 9 = 0$$

$$r^2 = -9$$

$$r = \pm \sqrt{-9} = \pm 3i$$

THE GENERAL SOLUTION IS  $y = C_1 \cos(3t) + C_2 \sin(3t)$

b) TO FIND A PARTICULAR SOLN TO THE NON-HOMOGENEOUS ODE, WE CONSIDER  $t \sin(3t)$  AND ITS DERIVATIVES:

$$y_p = At \sin(3t) + Bt \cos(3t)$$

$$\frac{dy_p}{dt} = A \sin(3t) + 3At \cos(3t) + B \cos(3t) - 3Bt \sin(3t)$$

$$\frac{d^2 y_p}{dt^2} = 6A \cos(3t) - 9At \sin(3t) - 6B \sin(3t) - 9Bt \cos(3t)$$

SUBSTITUTING INTO THE ODE, WE GET

$$\begin{aligned} & [6A \cos(3t) - 9At \sin(3t) - 6B \sin(3t) - 9Bt \cos(3t)] \\ & + 9[At \sin(3t) + Bt \cos(3t)] = \sin(3t) \end{aligned}$$

$$6A \cos(3t) - 6B \sin(3t) = \sin(3t)$$

$$6A = 0 \rightarrow A = 0 \quad -6B = 1 \rightarrow B = -\frac{1}{6}$$

$$\text{THUS } y_p = -\frac{1}{6} t \cos(3t)$$

SO THE GENERAL SOLN IS

$$y = y_c + y_p$$

$$= C_1 \cos(3t) + C_2 \sin(3t) - \frac{1}{6} t \cos(3t)$$