

1. b) THIS ODE IS SEPARABLE:

$$\frac{dy}{dt} = \cos^3(t) \cos^2(4y)$$

$$\frac{1}{\cos^2(4y)} dy = \cos^3(t) dt$$

$$\int \sec^2(4y) dy = \int \cos^3(t) dt$$

$$\frac{1}{4} \tan(4y) = \int [1 - \sin^2(t)] \cos(t) dt$$

$$u = \sin(t) \\ du = \cos(t) dt$$

$$= \int [1 - u^2] du$$

$$= u - \frac{1}{3} u^3 + C$$

$$= \sin(t) - \frac{1}{3} \sin^3(t) + C$$

$$\tan(4y) = 4 \sin(t) - \frac{4}{3} \sin^3(t) + C$$

SINCE  $y\left(\frac{\pi}{6}\right) = \frac{\pi}{16}$ , WE HAVE

$$\tan\left(\frac{\pi}{4}\right) = 4 \sin\left(\frac{\pi}{6}\right) - \frac{4}{3} \sin^3\left(\frac{\pi}{6}\right) + C$$

$$1 = 4 \cdot \frac{1}{2} - \frac{4}{3} \cdot \frac{1}{8} + C$$

$$1 = 2 - \frac{1}{6} + C$$

$$C = -\frac{5}{6}$$

THUS  $\tan(4y) = 4 \sin(t) - \frac{4}{3} \sin^3(t) - \frac{5}{6}$

$$y = \frac{1}{4} \arctan\left(4 \sin(t) - \frac{4}{3} \sin^3(t) - \frac{5}{6}\right)$$