1. a) THIS IS A BERNOULLI EQN WITH $n=3$

$$t\frac{dy}{dt} - y = \cos(t)y^3$$

$$ty^{-3}\frac{dy}{dt} - y^{-2} = \cos(t)$$

WE LET $v = y^{-2}$

$$\frac{dv}{dt} = -2y^{-3}\frac{dy}{dt}$$

$$-\frac{1}{2}\frac{dv}{dt} = y^{-3}\frac{dy}{dt}$$

THE ODE BECOMES

$$t\left(-\frac{1}{2}\frac{dv}{dt}\right) - v = \cos(t)$$

$$-\frac{1}{2}t\frac{dv}{dt} - v = \cos(t)$$

$$\frac{dv}{dt} + \frac{2}{t}v = \frac{2\cos(t)}{t}$$

$$p(t) = \frac{2}{t} \qquad g(t) = \frac{2\cos(t)}{t}$$

WE APPLY THE METHOD OF INTEGRATING FACTORS:

$$\mu = e^{\int p(t)} = e^{\int \frac{2}{t}dt} = e^{2\ln(t)} = e^{\ln(t^2)} = t^2$$

NOW WE REWRITE THE ODE AS

$$t^2\frac{dv}{dt} + 2tv = 2t\cos(t)$$

$$\frac{d}{dt}\left[t^2v\right] = 2t\cos(t)$$

$$t^2v = \int 2t\cos(t)dt$$

WE USE INTEGRATION BY PARTS WITH

$$w = 2t \qquad dv = \cos(t)dt$$

$$dw = 2dt \qquad v = \sin(t)$$

SO

$$t^2 v = 2t\sin(t) - \int 2\sin(t)dt$$

$$= 2t\sin(t) + 2\cos(t) + C$$

$$v = 2t^{-1}\sin(t) + 2t^{-2}\cos(t) + Ct^{-2}$$

$$y^{-2} = 2t^{-1}\sin(t) + 2t^{-2}\cos(t) + Ct^{-2}$$