

$$12. \quad \mathcal{L}\left\{\frac{d^2y}{dt^2} + 4y\right\} = \mathcal{L}\{0\}$$

$$\mathcal{L}\left\{\frac{d^2y}{dt^2}\right\} + 4\mathcal{L}\{y\} = 0$$

$$[s^2\mathcal{L}\{y\} - sy(0) - y'(0)] + 4\mathcal{L}\{y\} = 0$$

$$s^2\mathcal{L}\{y\} - 3s - 1 + 4\mathcal{L}\{y\} = 0$$

$$s^2\mathcal{L}\{y\} + 4\mathcal{L}\{y\} = 3s + 1$$

$$(s^2 + 4)\mathcal{L}\{y\} = 3s + 1$$

$$\mathcal{L}\{y\} = \frac{3s + 1}{s^2 + 4}$$

$$= 3 \cdot \frac{s}{s^2 + 4} + \frac{1}{2} \cdot \frac{2}{s^2 + 4}$$

$$= 3\mathcal{L}\{\cos(2t)\} + \frac{1}{2}\mathcal{L}\{\sin(2t)\}$$

$$\boxed{y = 3\cos(2t) + \frac{1}{2}\sin(2t)}$$

NOTE

$$\mathcal{L}\{\sin(2t)\} = \frac{2}{s^2 + 4}$$

$$\mathcal{L}\{\cos(2t)\} = \frac{s}{s^2 + 4}$$