Please remember that each final exam is different, and the style and emphasis of the questions will vary from semester to semester.

1. Solve each of the following equations.

[6] (a) $t\dfrac{dy}{dt} - y = \cos(t)y^3$

[7] (b) $\dfrac{dy}{dt} = \cos^3(t)\cos^2(4y), \quad y\left(\dfrac{\pi}{6}\right) = \dfrac{\pi}{16}$

[4] 2. Give the interval of definition of the solution to the initial value problem

$$(2-t)\frac{dy}{dt} - y = \tan(t), \quad y(3) = 0.$$

You do **NOT** need to solve the equation.

[10] 3. Show that the equation

$$1 - \left(y - \frac{t}{y}\right)\frac{dy}{dt} = 0$$

is not exact. Find an integrating factor which makes it exact, then solve the equation.

[7] 4. Identify the fixed points of the autonomous equation

$$\frac{dy}{dt} = 9y^2 - y^3 - 14y.$$

Determine, with justification, the stability of each fixed point. What is the long-term behaviour of the solution if $y(0) = 1$? If $y(0) = 2$? If $y(0) = 3$?

[4] 5. Show that $y_1 = t^2$ and $y_2 = t^{-2}$ form a fundamental set of solutions.

[10] 6. Use the method of reduction of order to find the general solution of

$$t^2\frac{d^2y}{dt^2} + 3t\frac{dy}{dt} + y = 0,$$

given that $y = \dfrac{1}{t}$ is a particular solution.

[3] 7. (a) Solve the homogeneous equation

$$\frac{d^2y}{dt^2} + 9y = 0.$$

[7] (b) Use the method of undetermined coefficients and your answer to part (a) to find the general solution of

$$\frac{d^2y}{dt^2} + 9y = \sin(3t).$$

[10] 8. Use the method of variation of parameters to solve

$$t^2\frac{d^2y}{dt^2} - t\frac{dy}{dt} + y = \ln(t),$$

given that $y_1 = t$ and $y_2 = t\ln(t)$ are solutions of the corresponding homogeneous equation.

[6] 9. Solve the equation

$$\frac{d^6y}{dt^6} - 5\frac{d^5y}{dt^5} + 4\frac{d^4y}{dt^4} - 20\frac{d^3y}{dt^3} = 0.$$

[4] 10. Use the definition of the Laplace transform to derive $\mathcal{L}\{e^{2t}\}$ (for $s > 2$).

11. Determine each of the following without directly using the definition of the Laplace transform.

[4] (a) $\mathcal{L}\{e^{2t}(t-2)\}$

[4] (b) $\mathcal{L}\{u_2(t)(t-2)\}$

[8] 12. Use the Laplace transform to solve the initial value problem

$$\frac{d^2y}{dt^2} + 4y = 0, \quad y(0) = 3, \quad y'(0) = 1.$$

[6] 13. An *Euler equation* is a second-order equation of the form

$$t^2\frac{d^2y}{dt^2} + At\frac{dy}{dt} + By = 0$$

for constants A and B. Show that the substitution $v = \ln(t)$ transforms an Euler equation into an equation with constant coefficients. *(Hint: Use the Chain Rule to effect the change of variables.)*