

## Mathematics 2130

### Project 3B

### Cutthroat

Trick-taking card games are common around the world. Bridge, for example, is internationally popular, while 120s has been a perennial favourite at the kitchen tables of Newfoundland and Labrador for many decades. In this project, we will consider a trick-taking game called **Cutthroat** and use mathematics to study the strategy which players should employ to give themselves the greatest chance of success in the game.

### The Rules of Cutthroat

Players are each dealt one card, and then a second, and so on, until a point is reached at which the number of cards remaining in the deck is insufficient to give each player a card. These cards are set aside (without being turned over) until the next hand. Beginning with the player to the left of the dealer and continuing clockwise, each player bids the number of **tricks** they think they will win in that hand; a trick is one round of each player playing one card. The sum of the bids is not allowed to equal the number of possible tricks; essentially, this is only a restriction on the dealer (who is last to bid).

For example, suppose a game is being played with five players. The players are each initially dealt 10 cards; the remaining two cards are set aside. The total of the five players' bid is not permitted to equal 10. Suppose the first player to the dealer's left bids 3 tricks, the second player bids 1 trick, and the third player bids 0 tricks. If the fourth player bids 5 tricks then the dealer would be permitted any bid *except* 1 trick. If the fourth player bids 6 tricks then the dealer would not be permitted to bid 0 tricks (but could bid any positive number of tricks). If the fourth player bids 7 tricks (or more) then the dealer would be permitted any bid, since it would not then be possible for the total bid to equal 10.

Cards in each suit are ranked from deuce (low) to ace (high). For each hand, either a trump suit is declared (one of Clubs, Diamonds, Hearts, or Spades) or the hand has "no trump." The first hand of the game has Spades as the trump suit. The next hand has Hearts, the next Clubs, then Diamonds, then "no trump". The sixth hand then reverts to Spades and the pattern is repeated.

Beginning with the player to the dealer's left and continuing clockwise around the table, each player takes a turn playing a card. The first player may play a card from the trump suit only if all of their remaining cards are trumps, or if trumps have been **broken** — that is, if a trump has already been played on a previous trick that hand. Players must **follow suit**, meaning that the card they play must match the suit of the first player's card if possible; if they do not have such a card remaining, then they may play any card they like (including a trump card). The winner of the trick is the player who has either played the highest card of the same suit as the first player, or the player who has played the highest trump; a trump always beats a non-trump. Then the winner of that trick plays the first card to start another trick, and the hand continues until all cards have been played.

Each hand in Cutthroat is scored separately. If a player has taken exactly the number of tricks as they bid, then that player scores 5 points plus a number of points equal to their bid. Otherwise, the player scores no points. The name of the game derives from this “cutthroat” scoring policy. In most trick-taking games, a player need only take as many tricks as were bid, and obtaining additional tricks is not penalised; in Cutthroat, that is not the case.

After each hand, the player to the left becomes the new dealer. For this next hand (and each subsequent hand), each player receives one card fewer than the previous hand, until finally each player receives only one card. After that, the number of cards each player receives is increased by one in each subsequent hand until finally a hand is played with the same number of cards as the first hand. After that hand, the game is over and the player with the highest total score is the winner.

## Example

Suppose Graham, Ryan and Yasmin are playing a hand in which Spades are the trump suit. The players each have two cards remaining, and trumps have not yet been broken. Graham won the last trick, so he lays the first card. In his hand he has the 7 of Diamonds and the Queen of Spades. Because trumps have not been broken, he cannot choose to play a Spade, so he is forced to play the 7 of Diamonds. Ryan is to Graham’s left, so he plays next. His remaining cards are the 3 of Hearts and the 2 of Diamonds. Since he must follow suit if possible, Ryan is required to play the 2 of Diamonds. Because this card is of the same suit as Graham’s, but of a lower rank, Graham is still winning the trick. Yasmin plays last. She has the 5 of Spades and the 10 of Clubs. Since she does not have a Diamond, she can play either card. If she wanted to win the trick, she could play the 5 of Spades; even though it is of lower rank than Graham’s 7 of Diamonds, it would out-count it by virtue of being a trump. On the other hand, if Yasmin did not want to win the trick (for example, if she had bid 0 or had already accumulated a number of tricks equal to her bid) then she could play the 10 of Clubs. Since it is neither a trump nor of the same suit as the first card played, it cannot win the trick regardless of its rank.

## Objectives

In this project, we are principally interested in the one-card hand, in which the only possible bids are 0 tricks or 1 trick. Suppose there are  $n$  Cutthroat players. What bidding strategy should be employed by the first player? That is, under what circumstances should the first player choose to bid 0 or choose to bid 1? Then, how should the second player bid? How should the  $i$ th player bid? Should the  $n$ th player (the dealer) ever bid 1 by choice, rather than being forced to do so in the situation where exactly one other player bid 1?

Players don’t always employ an optimal bidding strategy. For instance, it might be well-known that one of the players always bids 1 if they have a trump or 0 if they don’t. Or maybe one player always flips a coin to determine their bid. Do these types of behaviour change the bidding strategy for the other players?

Examine as many of these situations as you can, along with any other interesting questions regarding the one-card hand that you can think of. Then, feel free to consider what can be said about the two-card hand, or any other aspect of the game of Cutthroat.