

Mathematics 2130

Project 4B

Sequences and Switches

Consider a binary sequence consisting of m zeroes and n ones in any order. We call an element in the sequence a “switch” if it differs from the previous element. For instance, if $m = n = 3$, the sequence

$$\{0, 0, 1, 1, 0, 1\}$$

has three switches: the third, fifth and sixth elements.

Within this simple framework, we can ask many questions. Most basically, what is the total number of binary sequences with m zeroes and n ones? What is the minimum number of switches such a sequence can have? What is the maximum number? Most importantly, considering all possible sequences, what is the average number of switches?

Of course, there is nothing really special about binary sequences. A **k -ary sequence** is one in which every element is an integer from $0, 1, \dots, k - 1$. For example,

$$\{2, 0, 1, 1, 3, 3, 3, 3, 0, 0\}$$

is a 4-ary sequence (for which the second, third, fifth and ninth elements are switches). What can you say about switches when you consider k -ary sequences?

Prepare a report that addresses the issues mentioned above. As always, supporting mathematical reasoning is essential.