Mathematics 2130
Project 3
Quadratic and Cubic Equations via Maple

In high school, you learned to use the quadratic formula to solve

$$ax^2 + bx + c = 0. \tag{1}$$

This might seem to be the end of the story for quadratic equations, but in fact there are still a few things to be said about them, and in this project you will develop tools to study some of those properties. You will then use those tools to examine a cubic equation.

For the quadratic equation, the approach will be to view Equation (1) as implicitly defining a multivalued function where x is the dependent variable and (a, b, c) are the independent variables. In other words, for specific inputs of the coefficients, the value (or values) of the implicit function is defined to be the solution (or solutions) of the corresponding quadratic equation. You will investigate this implicit function by combining analytical work with visualisations generated by MAPLE.

First, simplify the problem by setting $a = 1$; your report should explain why this can be done without any significant loss of generality. Next, consider cross-sections of the resulting two-dimensional multivalued function — that is, pick a value of b and then plot x as a function of c (or vice versa). You should do this in three ways, as described below; this is partly because, in science, it's always desirable to find multiple ways of doing things and then compare the outcomes, but also because we want you to practise using MAPLE.

1. MAPLE has commands such as `solve` and `fsolve` which can be used to automatically solve many equations. Write a short program that, for a given value of b, runs over a range of values for c (or vice versa) and solves for the corresponding values of x. Store these (c, x) values (or (b, x) values) and then, at the end of the run, plot them (probably using either `plot` or `pointplot`).

2. Use the quadratic formula along with the MAPLE command `plot` to plot the "top" and "bottom" parts of the function separately (but on the same set of axes).

3. Finally, here is the easiest way to graph the function: look up the `implicitplot` command and do everything with just one command.

All of these should be included in your report. The corresponding plots should be selected to show interesting cross-sections of the function for both constant b and constant c and their appearance optimized for clarity (check out `plot[options]`).

Next, you should plot the full three-dimensional function. (*Hint:* To find the easiest way, use MAPLE's Help feature to browse through the plotting commands.) Once you understand the general form of the function, you should go back to the equations and try to explain as much as you can about the properties that you have observed. For example, you should discuss the domains corresponding to the cases where Equation (1) possesses zero, one, or two solutions. A useful tool in this analysis will be the discriminant.

Finally, having dealt with the quadratics, go on to the cubic equation

$$x^3 + bx^2 + c = 0. \tag{2}$$

As before you should examine cross-sections as well as a full three-dimensional plot of the solution space. However, this time you may choose to use any method you like to graph the solutions. Try to explain features of the graphs analytically, in as much depth as you can.