MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

TEST 2 | **MATHEMATICS 2051** | NOVEMBER 8TH, 2007

Name	**MUN Number**

[10] 1. Determine whether the matrix

$$A = \begin{bmatrix} -5 & 0 & 14 \\ -3 & 1 & 7 \\ -3 & 0 & 8 \end{bmatrix}$$

is diagonalizable. If so, find an invertible matrix P and a diagonal matrix D such that $D = P^{-1}AP$.

[6] 2. Prove that if $A = kI$ for any scalar k then the only matrix similar to A is A itself.

[6] 3. Let V be the set of vectors $\left[\begin{smallmatrix} a \\ b \end{smallmatrix}\right]$ in $\mathbb{R}^2$ with the usual operation of vector addition, but scalar multiplication defined by

$$k \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 2ka \\ 2kb \end{bmatrix}.$$

Give two reasons why V is not a vector space.

[9] 4. Let $F[0,1]$ be the set of continuous functions on the interval $[0,1]$. Either prove that the set

$$U = \{f \in F[0,1] \mid f(0) = f(1)\}$$

is a subspace of $F[0,1]$ or explain why it is not.

[4] 5. After an extended lunch at the Breezeway, your professor comes to class claiming to have found two sets, X and Y, both of which he says are bases of $\mathbb{R}^4$. The set X, he says, contains only three vectors. The set Y contains five vectors. Briefly explain why your professor is wrong on both counts.

[8] 6. Consider

$$U = \{x^2 + 3, \quad x - 1, \quad 2x^2 + 3x\}$$

as a subset of P_2 (the space of polynomials of degree at most 2). Determine whether or not U is linearly independent. If it is not, express one of the vectors as the linear combination of the others.

[7] 7. The set of 2×2 skew-symmetric matrices, $U = \{A \mid A^T = -A\}$, is a subspace of M_{22}. Exhibit a basis of U and find its dimension.

[50]