MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

TEST 1 **MATHEMATICS 2051** October 11th, 2007

Name	**MUN Number**

1. Define each of the following terms clearly and concisely.

[3] (a) the **linear independence** of a set of vectors $\{\underline{x}_1, \ldots, \underline{x}_p\}$

[3] (b) the **span** of a set of vector $\{\underline{x}_1, \ldots, \underline{x}_p\}$

[3] (c) the **basis** of a subspace U

[3] (d) the **null space** of a matrix A

[6] 2. If $\{\underline{x}, \underline{y}, \underline{z}\}$ spans a subspace U and $\underline{w}$ is a vector in U, prove that $\{\underline{w}, \underline{x}, \underline{y}, \underline{z}\}$ also spans U.

[10] 3. Determine whether the vectors $\underset{\sim}{x}_1 = \begin{bmatrix} 1 \\ -4 \\ 7 \end{bmatrix}$, $\underset{\sim}{x}_2 = \begin{bmatrix} 0 \\ 2 \\ -3 \end{bmatrix}$ and $\underset{\sim}{x}_3 = \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$ are linearly independent. If they are not, express one vector as the linear combination of the others.

[7] 4. Prove that if A is an $m \times n$ matrix then $\text{null}(A)$ is a subspace of $\mathbb{R}^n$.

5. Consider the matrix

$$A = \begin{bmatrix} 1 & 4 & -3 & -8 \\ 0 & -1 & 2 & 4 \\ 1 & 1 & 3 & 4 \end{bmatrix}.$$

[7] (a) Find bases for the column space and row space of A, and determine $\text{rank}(A)$.

[5] (b) Find a basis for $\text{null}(A)$ and compute $\dim[\text{null}(A)]$.

[3] (c) Briefly explain why your answers to parts (a) and (b) corroborate a theorem given in class. (At least, they should if you did parts (a) and (b) correctly!)

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[50]