MEMORIAL UNIVERSITY OF NEWFOUNDLAND
DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 8 | **Mathematics 2051** | FALL 2007

Due: Thursday, November 22nd, 2007. SHOW ALL WORK.

1. Prove that if $\{\underline{f}_1, \ldots, \underline{f}_n\}$ is an orthogonal basis for $\mathbb{R}^n$ and $U = \text{span}\{\underline{f}_1, \ldots, \underline{f}_p\}$ then
$$U^\perp = \text{span}\{\underline{f}_{p+1}, \ldots, \underline{f}_n\}.$$

2. For each of the following matrices A, find an orthogonal matrix Q and a diagonal matrix D such that $D = Q^{-1}AQ$.

   (a) $A = \begin{bmatrix} 13 & 3 \\ 3 & 5 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 3 & 0 & 5 \\ 0 & -2 & 0 \\ 5 & 0 & 3 \end{bmatrix}$

   (c) $A = \begin{bmatrix} 1 & -4 & 2 \\ -4 & 1 & -2 \\ 2 & -2 & -2 \end{bmatrix}$

3. Prove that for any orthogonal matrix Q, $\det(Q) = \pm 1$.

4. Consider the matrix $A = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}$.

   (a) Show that A is positive definite.

   (b) Find the Cholesky factorization of A, that is, find an upper triangular matrix U such that $A = U^T U$.

5. Let $\underline{x}$ and $\underline{y}$ be vectors in $\mathbb{C}^n$ such that $\|\underline{x}\| = 2$, $\|\underline{y}\| = 5$ and $\langle \underline{x}, \underline{y} \rangle = -5$. Determine $\|4\underline{x} + 3\underline{y}\|$.

6. Determine whether each function T is a linear transformation.

   (a) $T : M_{22} \to \mathbb{R}^3$ defined by
$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} a - d \\ 2b \\ c \end{bmatrix}$$

   (b) $T : P_2 \to P_2$ defined by
$$T(a + bx + cx^2) = (a+1) + (b+1)x + (c+1)x^2$$

   (c) $T : M_{nn} \to \mathbb{R}$ defined by $T(A) = \det(A)$

   (d) $T : P_2 \to P_3$ defined by $T(p(x)) = xp(x)$