MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 7 | **Mathematics 2051** | FALL 2007

Due: Thursday, November 15th, 2007. SHOW ALL WORK.

1. Let U be a subspace of $\mathbb{R}^4$ spanned by the vectors

$$\underline{x}_1 = \begin{bmatrix} 1 \\ -3 \\ 0 \\ 2 \end{bmatrix}, \quad \underline{x}_2 = \begin{bmatrix} 2 \\ 0 \\ -4 \\ -1 \end{bmatrix}, \quad \underline{x}_3 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -2 \end{bmatrix}.$$

   (a) Show that $\{\underline{x}_1, \underline{x}_2, \underline{x}_3\}$ is an orthogonal basis for U.

   (b) Find an orthonormal basis of U.

   (c) The vector

$$\underline{y} = \begin{bmatrix} 5 \\ -3 \\ 2 \\ -10 \end{bmatrix}$$

   lies in U. Express $\underline{y}$ as a linear combination of the orthogonal basis vectors $\underline{x}_1$, $\underline{x}_2$, $\underline{x}_3$.

   (d) The vector

$$\underline{z} = \begin{bmatrix} -2 \\ -2 \\ 0 \\ 5 \end{bmatrix}$$

   does not lie in U. Find the projection $\underline{p} = \text{proj}_U(\underline{z})$ of $\underline{z}$ onto U. Write $\underline{z}$ as the sum of a vector in U and a vector in $U^\perp$.

   (e) Find a basis for $U^\perp$ and state $\dim(U^\perp)$.

2. Let $\underline{x}$ and $\underline{y}$ be vectors such that $\|\underline{x}\| = 2$, $\|\underline{y}\| = 5$ and $\underline{x} \cdot \underline{y} = -5$. Determine $\|4\underline{x} + 3\underline{y}\|$.

3. Use the Gram-Schmidt algorithm to find an orthogonal basis for $\mathbb{R}^3$ using the basis vectors

$$\underline{x}_1 = \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}, \quad \underline{x}_2 = \begin{bmatrix} -2 \\ -3 \\ 4 \end{bmatrix}, \quad \underline{x}_3 = \begin{bmatrix} 4 \\ 0 \\ -4 \end{bmatrix}.$$

PLEASE TURN OVER

4. Let U be the subspace of $\mathbb{R}^4$ spanned by the orthogonal vectors

$$\underline{x}_1 = \begin{bmatrix} -4 \\ -4 \\ 2 \\ 0 \end{bmatrix} \quad \text{and} \quad \underline{x}_2 = \begin{bmatrix} -1 \\ 0 \\ -2 \\ -1 \end{bmatrix}.$$

Find the vector in U closest to the vector $\underline{y} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$.