MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 6 **Mathematics 2051** FALL 2007

Due: Tuesday, October 30th, 2007. SHOW ALL WORK.

1. Determine which of the following sets U are linearly independent in the given vector space V.

   (a) $V = M_{22}$, $U = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \right\}$

   (b) $V = P_3$, $U = \{x^3 + 2x, 3 - x^3, 2x^2 - 2x + 1\}$

   (c) $V = F[-1, 0]$, $U = \left\{ \frac{1}{x^2 - 4}, \frac{1}{x^2 + x - 2}, \frac{1}{x^2 - 3x + 2} \right\}$

2. Consider two $n \times n$ matrices A and B. If A is symmetric (that is, $A = A^T$) and B is skew-symmetric (that is, $B = -B^T$), show that the set $\{A, B\}$ is linearly independent in M_{nn}.

3. (a) Let $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. Exhibit a basis for the subspace U of M_{22} defined by

   $$U = \{X \mid AX = XA\}$$

   and state $\dim(U)$.

   (b) Find a basis for the subspace U of P_3 given by

   $$U = \{p(x) \mid -p(x) = p(-x)\}$$

   (that is, the subspace of odd polynomials). What is $\dim(U)$?

4. Prove that any non-zero vector in a finite dimensional vector space must be part of a basis.

5. Let $\underline{y}$, $\underline{x}_1$, $\ldots$, $\underline{x}_n$ be vectors in a vector space V. If $U = \text{span}\{\underline{x}_1, \ldots, \underline{x}_n\}$ and $W = \text{span}\{\underline{y}, \underline{x}_1, \ldots, \underline{x}_n\}$, prove that either $\dim(W) = \dim(U)$ or $\dim(W) = \dim(U) + 1$.

6. Show that $F[a, b]$ is infinite-dimensional for any a and b. *(Hint: Consider specific types of functions which must be in $F[a, b]$, particularly those we have used to form other vector spaces.)*