MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 5 **Mathematics 2051** FALL 2007

Due: Tuesday, October 23rd, 2007. SHOW ALL WORK.

1. Prove that the zero vector in any vector space is unique.
2. Prove that if $\underline{x}$ is a non-zero vector in a vector space V and k and ℓ are scalars then $k\underline{x} = \ell\underline{x}$ implies $k = \ell$.
3. Consider each of the following vector spaces V and their subsets U. Either prove that U is a subspace of V, or explain why it is not.

   (a) $V = M_{22}$, the space of all 2×2 matrices; $U = \{A \mid A \in M_{22}, A = A^T\}$, the set of all symmetric 2×2 matrices

   (b) $V = F[0,1]$, the space of continuous functions on $[0,1]$; $U = \left\{ f \;\middle|\; \int_0^1 f(x)\,dx = 0 \right\}$

   (c) $V = F[0,1]$; $U = \left\{ f \;\middle|\; \int_0^1 f(x)\,dx = 1 \right\}$

   (d) $V = P_3$, the space of polynomials of degree at most 3; $U = \{xp(x) \mid p(x) \in P_2\}$

   (e) $V = P_3$; $U = \{xp(x) \mid p(x) \in P_3\}$
4. Determine whether $\underline{x} = 5x^2 - 6x + 7$ lies in the span of the vectors $\underline{u} = x^2 - 3$ and $\underline{v} = 3x + 4$.
5. Determine whether M_{22} is spanned by the vectors

$$\underline{x}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \underline{x}_2 = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}, \quad \underline{x}_3 = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}, \quad \underline{x}_4 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}.$$