MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 4 | **Mathematics 2051** | FALL 2007

Due: Tuesday, October 16th, 2007. SHOW ALL WORK.

1. For each of the following matrices A, determine whether A is diagonalizable. If so, find a diagonal matrix D and an invertible matrix P such that $D = P^{-1}AP$.

   (a) $A = \begin{bmatrix} 6 & 4 & -4 \\ -4 & -4 & 6 \\ 0 & 0 & 1 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 0 & 1 & -1 \\ 2 & 1 & -2 \\ -1 & -1 & 0 \end{bmatrix}$

   (c) $A = \begin{bmatrix} 0 & 2 & -1 \\ 1 & 1 & -1 \\ 3 & -2 & 0 \end{bmatrix}$

2. Consider two similar matrices A and B. Prove that if A is an *idempotent* matrix (that is, $A^2 = A$) then B is also idempotent.

3. Let λ be an eigenvalue of A with corresponding eigenvector $\underline{x}$. Show that if A and B are similar matrices such that $B = P^{-1}AP$ then λ is also an eigenvalue of B with corresponding eigenvector $P^{-1}\underline{x}$. (That is, show that $BP^{-1}\underline{x} = \lambda P^{-1}\underline{x}$.)

4. For each of the following sets, either prove that the set is a vector space with the indicated operations, or explain why it is not.

   (a) The set A of all 2×2 matrices of the form $\begin{bmatrix} x & x+y \\ x-y & y \end{bmatrix}$ with the usual operations of matrix addition and scalar multiplication

   (b) The set B of ordered pairs of real numbers (x, y) where $y \geq 0$, with the usual operations of vector addition and scalar multiplication

   (c) The set C of ordered triples of real numbers (x, y, z), with the usual operation of vector addition, but scalar multiplication defined to be

   $$k(x, y, z) = (kx, y, z)$$

   (d) The set D of ordered triples of real numbers (x, y, z), with the usual operation of vector addition, but scalar multiplication defined to be

   $$k(x, y, z) = (z, kx, y)$$

   (e) The set E of all continuous real-valued functions f such that $f(1) = 0$, with the usual operations of function addition and scalar multiplication