MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

Assignment 3	**Mathematics 2051**	Fall 2007

Due: Tuesday, October 2nd, 2007. SHOW ALL WORK.

1. Find bases for the column space and row space of the matrix A and determine rank(A), where
$$A = \begin{bmatrix} 1 & -3 & 2 & 5 \\ -3 & 3 & -1 & -9 \\ -2 & 0 & 1 & -4 \\ -4 & -6 & 7 & -2 \end{bmatrix}.$$

2. Find a basis for the subspace U given by
$$U = \text{span}\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \\ -4 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ 3 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -2 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ -1 \\ 3 \\ 3 \end{bmatrix} \right\}.$$

3. Consider the matrix
$$A = \begin{bmatrix} 1 & 4 & -2 \\ 0 & 3 & 1 \\ 1 & 1 & -3 \\ 2 & -1 & -7 \\ -2 & -5 & 5 \end{bmatrix}.$$
   (a) Find bases for the column space and row space of A and determine rank(A).
   (b) Find a basis for the null space of A and compute its dimension.
   (c) Do your answers to (a) and (b) make sense, given the size of the matrix A?

4. Prove that for matrices U and A, rank(UA) $\leq$ rank(A), with equality if U is invertible.

5. (a) Show that if matrices A and B have linearly independent columns then the columns of AB are also linearly independent.
   (b) Show that if matrices A and B have linearly independent rows then the rows of AB are also linearly independent.