MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 2 | **Mathematics 2051** | FALL 2007

Due: Tuesday, September 25th, 2007. SHOW ALL WORK.

1. Determine whether each of the following sets of vectors is linearly independent.

   (a) $\left\{ \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 6 \\ -4 \end{bmatrix} \right\}$

   (b) $\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ -3 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ -2 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \\ -4 \\ 9 \end{bmatrix} \right\}$

   (c) $\left\{ \begin{bmatrix} 2 \\ -3 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 5 \\ 4 \end{bmatrix}, \begin{bmatrix} 6 \\ -8 \\ 2 \\ 1 \end{bmatrix} \right\}$

2. Determine whether $\mathbb{R}^3$ is spanned by the vectors

$$\underline{x}_1 = \begin{bmatrix} 4 \\ 1 \\ -3 \end{bmatrix}, \quad \underline{x}_2 = \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}, \quad \underline{x}_3 = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}.$$

3. Show that $\text{span}\{\underline{x}, \underline{y}\} = \text{span}\{\underline{x} - \underline{y},\ 2\underline{x} + 3\underline{y}\}$ for any vectors $\underline{x}$ and $\underline{y}$ in $\mathbb{R}^n$, by proving the following.

   (a) Prove that $\text{span}\{\underline{x} - \underline{y},\ 2\underline{x} + 3\underline{y}\}$ is contained in $\text{span}\{\underline{x}, \underline{y}\}$.

   (b) Prove that $\text{span}\{\underline{x}, \underline{y}\}$ is contained in $\text{span}\{\underline{x} - \underline{y},\ 2\underline{x} + 3\underline{y}\}$.

   (Hint: For both (a) and (b), you should make use of a theorem we've proved in class which states that a set spanned by vectors is a subspace, and which also relates other subspaces to this subspace.)

4. Find a basis and calculate the dimension of each of the following subspaces of $\mathbb{R}^4$.

   (a) $U = \text{span}\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -3 \\ -2 \end{bmatrix} \right\}$

   (b) $U = \text{span}\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -3 \\ 5 \\ 2 \\ 0 \end{bmatrix} \right\}$

5. (a) Let $\{\underline{x}, \underline{y}, \underline{z}\}$ be a linearly independent set of vectors in $\mathbb{R}^4$. Prove that if $\underline{w}$ is not in $\text{span}\{\underline{x}, \underline{y}, \underline{z}\}$ then $\{\underline{x}, \underline{y}, \underline{z}, \underline{w}\}$ is also linearly independent.

   (b) Explain why the set $\{\underline{x}, \underline{y}, \underline{z}, \underline{w}\}$ found in (a) must be a basis for $\mathbb{R}^4$.