MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 1 | **Mathematics 2051** | FALL 2007

Due: Tuesday, September 18th, 2007. SHOW ALL WORK.

1. In each case, either prove that U is a subspace of $\mathbb{R}^3$, or explain why it is not.

   (a) $U = \left\{ \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \;\middle|\; x, y \in \mathbb{R} \right\}$

   (b) $U = \left\{ \begin{bmatrix} x \\ y^2 \\ z \end{bmatrix} \;\middle|\; x, y, z \in \mathbb{R} \right\}$

   (c) $U = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \;\middle|\; 2x + 3y = 4z, \quad x, y, z \in \mathbb{R} \right\}$

   (d) $U = \left\{ \begin{bmatrix} x+y \\ y+z \\ z+x \end{bmatrix} \;\middle|\; x, y, z \in \mathbb{R} \right\}$

2. Prove that if A is an $m \times n$ matrix and U is an invertible $m \times m$ matrix then $\text{null}(A) = \text{null}(UA)$. *(Hint: show that any element in* $\text{null}(A)$ *must also be in* $\text{null}(UA)$*, and vice versa.)*

3. Given the vectors

$$\underline{x} = \begin{bmatrix} 1 \\ 3 \\ -7 \\ -2 \end{bmatrix}, \quad \underline{y} = \begin{bmatrix} 1 \\ 1 \\ -3 \\ 1 \end{bmatrix}, \quad \underline{u} = \begin{bmatrix} -6 \\ 0 \\ 6 \\ -5 \end{bmatrix}, \quad \underline{v} = \begin{bmatrix} 2 \\ -4 \\ 6 \\ 11 \end{bmatrix},$$

   let $U = \text{span}\{\underline{x}, \underline{y}\}$ and determine if $\underline{u}$ and $\underline{v}$ are in U. If so, write that vector as the linear combination of $\underline{x}$ and $\underline{y}$.

4. Let

$$U = \left\{ \begin{bmatrix} 0 \\ x \\ y \end{bmatrix} \;\middle|\; x, y \in \mathbb{R} \right\}.$$

   Explain why U cannot be spanned by the vectors $\underline{x} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ and $\underline{y} = \begin{bmatrix} -1 \\ 3 \\ 4 \end{bmatrix}$.