MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 3.1 **Math 2050 Worksheet** WINTER 2018

SOLUTIONS

1. (a) (i) $M = \begin{bmatrix} 3 & 9 & -10 \\ 19 & -4 & 18 \\ -1 & -3 & -17 \end{bmatrix}$

   (ii) $C = \begin{bmatrix} 3 & -9 & -10 \\ -19 & -4 & -18 \\ -1 & 3 & -17 \end{bmatrix}$

   (iii) $AC^T = \begin{bmatrix} 2 & -5 & -1 \\ -3 & -1 & 0 \\ 2 & 4 & -3 \end{bmatrix} \begin{bmatrix} 3 & -19 & -1 \\ -9 & -4 & 3 \\ -10 & -18 & -17 \end{bmatrix} = 61I$ so $\det A = 61$

   (iv) $A^{-1} = \dfrac{1}{\det A} C^T = \dfrac{1}{61} \begin{bmatrix} 3 & -19 & -1 \\ -9 & -4 & 3 \\ -10 & -18 & -17 \end{bmatrix}$

   (b) (i) $M = \begin{bmatrix} 6 & -3 \\ -8 & 4 \end{bmatrix}$

   (ii) $C = \begin{bmatrix} 6 & 3 \\ 8 & 4 \end{bmatrix}$

   (iii) $AC^T = \begin{bmatrix} 4 & -8 \\ -3 & 6 \end{bmatrix} \begin{bmatrix} 6 & 8 \\ 3 & 4 \end{bmatrix} = 0$ so $\det A = 0$

   (iv) Since $\det A = 0$, A is not invertible.

2. (a) We expand along the first row:

$$\det A = 4 \begin{vmatrix} -2 & -5 \\ 1 & 3 \end{vmatrix} + \begin{vmatrix} -2 & -2 \\ 9 & 1 \end{vmatrix} = 4(-1) + 16 = 12.$$

   (b) We expand along the second row:

$$\det B = 5 \begin{vmatrix} 1 & -3 & 4 \\ -1 & 1 & -1 \\ -4 & 2 & 1 \end{vmatrix} - \begin{vmatrix} 1 & -3 & 4 \\ -1 & 0 & -1 \\ -4 & 4 & 1 \end{vmatrix}.$$

   To compute the first of these 3×3 determinants, we expand along the first row:

$$\begin{vmatrix} 1 & -3 & 4 \\ -1 & 1 & -1 \\ -4 & 2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} - (-3) \begin{vmatrix} -1 & -1 \\ -4 & 1 \end{vmatrix} + 4 \begin{vmatrix} -1 & 1 \\ -4 & 2 \end{vmatrix} = 3 + 3(-5) + 4(2) = -4.$$

   To compute the second 3×3 determinant, we expand along the second row:

$$\begin{vmatrix} 1 & -3 & 4 \\ -1 & 0 & -1 \\ -4 & 4 & 1 \end{vmatrix} = -(-1) \begin{vmatrix} -3 & 4 \\ 4 & 1 \end{vmatrix} - (-1) \begin{vmatrix} 1 & -3 \\ -4 & 4 \end{vmatrix} = -19 + (-8) = -27.$$

   Thus

$$\det B = 5(-4) - (-27) = 7.$$

(c) We expand along the fourth column:

$$\det C = -(-5)\begin{vmatrix} 1 & 1 & -2 \\ 0 & -3 & 4 \\ 3 & 3 & 2 \end{vmatrix}.$$

To evaluate this determinant we expand along the second row:

$$\begin{vmatrix} 1 & 1 & -2 \\ 0 & -3 & 4 \\ 3 & 3 & 2 \end{vmatrix} = -3\begin{vmatrix} 1 & -2 \\ 3 & 2 \end{vmatrix} - 4\begin{vmatrix} 1 & 1 \\ 3 & 3 \end{vmatrix} = -3(8) - 4(0) = -24.$$

Thus

$$\det C = 5(-24) = -120.$$