SOLUTIONS

1. (a) Observe that $ad - bc = -6 - (-5) - 1 \neq 0$ so this matrix is invertible:

$$A^{-1} = \frac{1}{-1}\begin{bmatrix} -3 & 5 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ 1 & -2 \end{bmatrix}.$$

(b) This time $ad - bc = 0 - 6 = -6 \neq 0$ so this matrix is also invertible:

$$B^{-1} = \frac{1}{-6}\begin{bmatrix} 6 & 3 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} -1 & -\frac{1}{2} \\ -\frac{1}{3} & 0 \end{bmatrix}.$$

(c) In this case, $ad - bc = -9 - (-9) = 0$ so C is not invertible.

2. (a) Observe that

$$AB = \begin{bmatrix} -6 & 8 & 9 \\ 1 & -1 & -1 \\ -3 & 4 & 5 \end{bmatrix}\begin{bmatrix} 1 & 4 & -1 \\ 2 & 3 & -3 \\ -1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I,$$

so $B = A^{-1}$.

(b) A and B are not square matrices, so therefore they are not invertible. Alternatively, we could observe that while $AB = I$,

$$BA = \begin{bmatrix} \frac{7}{2} & \frac{15}{2} & 15 \\ \frac{5}{6} & \frac{7}{2} & 5 \\ -1 & -3 & -5 \end{bmatrix} \neq I.$$

3. Multiplying both sides by X gives

$$\begin{aligned} (A + BX^{-1})X &= (CX^{-1})X \\ AX + BX^{-1}X &= CX^{-1}X \\ AX + BI &= CI \\ AX + B &= C \\ AX &= C - B \\ X &= A^{-1}(C - B), \end{aligned}$$

where we have used the fact that A is invertible.

4. Given two matrices, we know that $(AB)^T = B^T A^T$. So let $B = YZ$ and observe that $B^T = (YZ)^T = Z^T Y^T$. Then

$$(XYZ)^T = (XB)^T = B^T X^T = Z^T Y^T X^T.$$