MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

For practice only. Not to be submitted.

1. Solve each of the following homogeneous systems of equations using Gaussian elimination and back-substitution. If a solution exists, express it as a vector or as a linear combination of vectors.

(a) $$\left.\begin{array}{rcrcrcl} a & - & 2b & - & 2c & = & 0 \\ -4a & + & 8b & + & 6c & = & 0 \end{array}\right\}$$

(b) $$\left.\begin{array}{rcrcrcrcl} x_1 & - & 3x_2 & & & + & 4x_4 & = & 0 \\ -x_1 & + & x_2 & + & 4x_3 & - & 2x_4 & = & 0 \\ x_1 & & & - & 6x_3 & + & x_4 & = & 0 \\ 2x_1 & - & 5x_2 & - & 2x_3 & + & 7x_4 & = & 0 \end{array}\right\}$$

2. Using your answers to #1, show that solutions to the following systems of equations can be written in the form $\mathbf{x} = \mathbf{x}_p + \mathbf{x}_h$, where $\mathbf{x}_p$ is a particular solution of the given system and $\mathbf{x}_h$ is a solution of the corresponding homogeneous system.

(a) $$\left.\begin{array}{rcrcrcr} a & - & 2b & - & 2c & = & -5 \\ -4a & + & 8b & + & 6c & = & 9 \end{array}\right\}$$

(b) $$\left.\begin{array}{rcrcrcrcl} x_1 & - & 3x_2 & & & + & 4x_4 & = & 6 \\ -x_1 & + & x_2 & + & 4x_3 & - & 2x_4 & = & -8 \\ x_1 & & & - & 6x_3 & + & x_4 & = & 9 \\ 2x_1 & - & 5x_2 & - & 2x_3 & + & 7x_4 & = & 13 \end{array}\right\}$$

3. Use Gaussian elimination and back-substitution to determine whether each of the following sets of vectors is linearly independent or linearly dependent.

(a) $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 3 \\ -4 \\ -1 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 5 \\ -3 \\ -1 \end{bmatrix}$

(b) $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 7 \\ -5 \\ -6 \\ 0 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 2 \\ 5 \\ -1 \\ 0 \\ 3 \end{bmatrix}$

(c) $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ -1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 4 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 3 \\ 4 \\ 1 \\ -1 \end{bmatrix}$, $\mathbf{v}_4 = \begin{bmatrix} 6 \\ 4 \\ 0 \\ -4 \end{bmatrix}$

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4. Prove that if A and B are suitably-sized matrices with linearly independent columns then their product AB also has linearly independent columns.