

MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

TEST 1

MATHEMATICS 2050

FEBRUARY 16TH, 2026

Name

MUN Number

1. Consider the vectors $\mathbf{u} = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 4 \\ 1 \\ -5 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} 6 \\ -1 \\ 0 \end{bmatrix}$.

[3] (a) Determine a vector of length 10 in the same direction as \mathbf{u} .

[5] (b) Find the equation of the plane spanned by \mathbf{u} and \mathbf{v} that passes through the point $(2, -1, 7)$.

[7] (c) Determine whether \mathbf{u} , \mathbf{v} and \mathbf{w} are linearly independent or linearly dependent. If they are linearly dependent, express one of the vectors as the linear combination of the other vectors.

2. Let ℓ be the line with vector equation

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}.$$

[4] (a) Find the cosine of the angle between ℓ and the line with vector equation

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ -4 \\ 5 \end{bmatrix} + t \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}.$$

[4] (b) Find the point at which ℓ intersects with the plane $4x - 2y + z = 8$, or explain why no such point of intersection can exist.

[7] (c) Find the distance from the point $P(2, -1, -1)$ to ℓ .

[4] 3. Determine all values of x for which the vectors $\mathbf{u} = \begin{bmatrix} x \\ x \\ 1 \\ 1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} x \\ 4 \\ -x \\ 2 \end{bmatrix}$ are orthogonal.

[6] 4. (a) Define what it means for vectors \mathbf{e} and \mathbf{f} to span a plane π .

(b) Define what it means for vectors \mathbf{u} and \mathbf{v} to be orthogonal.

(c) Suppose π is a plane spanned by vector \mathbf{e} and \mathbf{f} , and \mathbf{u} is a vector that is orthogonal to both \mathbf{e} and \mathbf{f} . Prove that \mathbf{u} must be orthogonal to every vector \mathbf{v} in π .