MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

Assignment 7	**MATHEMATICS 2050**	Winter 2026

SOLUTIONS

[3] 1. We bring A to row-echelon form using only the third elementary row operation:

$$\begin{bmatrix} -2 & 3 & 1 \\ 1 & 3 & -3 \\ -6 & 0 & -2 \end{bmatrix} \xrightarrow[R_3 \to R_3 - 3R_1]{R_2 \to R_2 - \left(-\frac{1}{2}\right)R_1} \begin{bmatrix} -2 & 3 & 1 \\ 0 & \frac{9}{2} & -\frac{5}{2} \\ 0 & -9 & -5 \end{bmatrix} \xrightarrow{R_3 \to R_3 - (-2)R_2} \begin{bmatrix} -2 & 3 & 1 \\ 0 & \frac{9}{2} & -\frac{5}{2} \\ 0 & 0 & -10 \end{bmatrix}.$$

Thus $A = LU$ where

$$\boxed{L = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ 3 & -2 & 1 \end{bmatrix}} \quad \text{and} \quad \boxed{U = \begin{bmatrix} -2 & 3 & 1 \\ 0 & \frac{9}{2} & -\frac{5}{2} \\ 0 & 0 & -10 \end{bmatrix}}.$$

2. Observe that

$$m_{11} = \begin{vmatrix} -5 & -1 \\ -1 & 0 \end{vmatrix} = -1 \implies c_{11} = m_{11} = -1$$

$$m_{12} = \begin{vmatrix} 3 & -1 \\ 2 & 0 \end{vmatrix} = 2 \implies c_{12} = (-1)m_{12} = -2$$

$$m_{13} = \begin{vmatrix} 3 & -5 \\ 2 & -1 \end{vmatrix} = 7 \implies c_{13} = m_{13} = 7$$

$$m_{21} = \begin{vmatrix} 0 & 3 \\ -1 & 0 \end{vmatrix} = 3 \implies c_{21} = (-1)m_{21} = -3$$

$$m_{22} = \begin{vmatrix} 4 & 3 \\ 2 & 0 \end{vmatrix} = -6 \implies c_{22} = m_{22} = -6$$

$$m_{23} = \begin{vmatrix} 4 & 0 \\ 2 & -1 \end{vmatrix} = -4 \implies c_{23} = (-1)m_{23} = 4$$

$$m_{31} = \begin{vmatrix} 0 & 3 \\ -5 & -1 \end{vmatrix} = 15 \implies c_{31} = m_{31} = 15$$

$$m_{32} = \begin{vmatrix} 4 & 3 \\ 3 & -1 \end{vmatrix} = -13 \implies c_{32} = (-1)m_{32} = 13$$

$$m_{33} = \begin{vmatrix} 4 & 0 \\ 3 & -5 \end{vmatrix} = -20 \implies c_{33} = m_{33} = -20.$$

[3] (a) From the above, $\boxed{M = \begin{bmatrix} -1 & 2 & 7 \\ 3 & -6 & -4 \\ 15 & -13 & -20 \end{bmatrix}}$.

[1] (b) From the above, $C = \begin{bmatrix} -1 & -2 & 7 \\ -3 & -6 & 4 \\ 15 & 13 & -20 \end{bmatrix}$.

[2] (c) We have

$$AC^T = \begin{bmatrix} 4 & 0 & 3 \\ 3 & -5 & -1 \\ 2 & -1 & 0 \end{bmatrix} \begin{bmatrix} -1 & -3 & 15 \\ -2 & -6 & 13 \\ 7 & 4 & -20 \end{bmatrix} = \begin{bmatrix} 17 & 0 & 0 \\ 0 & 17 & 0 \\ 0 & 0 & 17 \end{bmatrix}.$$

Hence $\det(A) = 17.$

[1] (d) Since $\det(A) \neq 0$, we know that A is invertible. Therefore

$$A^{-1} = \frac{1}{\det(A)} C^T = \frac{1}{17} \begin{bmatrix} -1 & -3 & 15 \\ -2 & -6 & 13 \\ 7 & 4 & -20 \end{bmatrix}.$$

[2] 3. (a) Expanding along the second row, we have

$$\det(A) = 5 \begin{vmatrix} 4 & -2 \\ 3 & -2 \end{vmatrix} + 0 - \begin{vmatrix} 1 & 4 \\ -1 & 3 \end{vmatrix} = 5(-2) - 7 = -17.$$

[6] (b) Expanding along the first row, we have

$$\det(B) = -2 \begin{vmatrix} 5 & -3 & 1 \\ 0 & -1 & 1 \\ -3 & 4 & -1 \end{vmatrix} - 6 \begin{vmatrix} 2 & -3 & 1 \\ 9 & -1 & 1 \\ 0 & 4 & -1 \end{vmatrix} + 0 - 0.$$

To evaluate the first of the 3×3 determinants, we expand along the second row:

$$\begin{vmatrix} 5 & -3 & 1 \\ 0 & -1 & 1 \\ -3 & 4 & -1 \end{vmatrix} = 0 + (-1) \begin{vmatrix} 5 & 1 \\ -3 & -1 \end{vmatrix} - \begin{vmatrix} 5 & -3 \\ -3 & 4 \end{vmatrix} = -(-2) - 11 = -9.$$

To compute the second of the 3×3 determinants, we expand along the third row:

$$\begin{vmatrix} 2 & -3 & 1 \\ 9 & -1 & 1 \\ 0 & 4 & -1 \end{vmatrix} = 0 - 4 \begin{vmatrix} 2 & 1 \\ 9 & 1 \end{vmatrix} + (-1) \begin{vmatrix} 2 & -3 \\ 9 & -1 \end{vmatrix} = -4(-7) - 25 = 3.$$

Hence

$$\det(B) = -2(-9) - 6(3) = 0.$$

[2] 4. First observe that

$$\det(U) = 1 \cdot 1 \cdot 1 \cdot 6 \cdot 7 = 42.$$

The only row operations which impact the determinant are the second and fourth operations, which introduce factors of -1 and $\frac{1}{2}$. Thus

$$\det(U) = -\frac{1}{2} \det(A) \quad \Longrightarrow \quad \det(A) = -2 \det(U) = -2 \cdot 42 = -84.$$