MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 4 | **MATHEMATICS 2050** | WINTER 2026

SOLUTIONS

[2] 1. (a) First,

$$AB = \begin{bmatrix} 5 & 2 & -1 & 3 \\ 0 & 7 & 7 & 1 \\ -4 & -1 & 0 & 5 \end{bmatrix} \begin{bmatrix} -2 & 0 & 0 \\ 1 & -1 & -2 \\ -2 & 0 & 6 \\ 0 & 9 & 4 \end{bmatrix} = \begin{bmatrix} -6 & 25 & 2 \\ -7 & 2 & 32 \\ 7 & 46 & 22 \end{bmatrix}.$$

[2] (b) Next,

$$BA = \begin{bmatrix} -2 & 0 & 0 \\ 1 & -1 & -2 \\ -2 & 0 & 6 \\ 0 & 9 & 4 \end{bmatrix} \begin{bmatrix} 5 & 2 & -1 & 3 \\ 0 & 7 & 7 & 1 \\ -4 & -1 & 0 & 5 \end{bmatrix} = \begin{bmatrix} -10 & -4 & 2 & -6 \\ 13 & -3 & -8 & -8 \\ -34 & -10 & 2 & 24 \\ -16 & 59 & 63 & 29 \end{bmatrix}.$$

[1] (c) We cannot compute $B^2 = BB$ because the first matrix in the product has 3 columns while the second matrix has four rows.

[4] 2. Observe that

$$A^2 = \begin{bmatrix} x & 6 \\ -1 & y \end{bmatrix} \begin{bmatrix} x & 6 \\ -1 & y \end{bmatrix} = \begin{bmatrix} x^2 - 6 & 6x + 6y \\ -x - y & -6 + y^2 \end{bmatrix}.$$

Thus we set

$$\begin{aligned} x &= x^2 - 6 \\ 6 &= 6x + 6y \\ -1 &= -x - y \\ y &= -6 + y^2. \end{aligned}$$

From the first equation, we have

$$x^2 - x - 6 = 0 \quad \Longrightarrow \quad (x-3)(x+2) = 0$$

and so $x = 3$ or $x = -2$. From the fourth equation, we obtain

$$y^2 - y - 6 = 0 \quad \Longrightarrow \quad (y-3)(y+2) = 0$$

so $y = 3$ or $y = -2$. From both the second and third equations, we get

$$x + y = 1.$$

Therefore A will be idempotent if $x = 3$ and $y = -2$ or if $x = -2$ and $y = 3$.

[3] 3. This system corresponds to the matrix equation $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{bmatrix} -1 & 5 \\ 2 & -7 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}.$$

Since

$$A^{-1} = \frac{1}{7-10} \begin{bmatrix} -7 & -5 \\ -2 & -1 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 7 & 5 \\ 2 & 1 \end{bmatrix}$$

the solution of the system is

$$\mathbf{x} = A^{-1}\mathbf{b} = \frac{1}{3} \begin{bmatrix} 7 & 5 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 6 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 51 \\ 12 \end{bmatrix} = \begin{bmatrix} 17 \\ 4 \end{bmatrix}$$

so $x = 17$ and $y = 4$.

[2] 4. This system corresponds to the matrix equation $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{bmatrix} 3 & -6 \\ -2 & 4 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 12 \\ -8 \end{bmatrix}.$$

However A is not invertible because $ad - bc = 12 - 12 = 0$ so the matrix inverse cannot be used here.

[3] 5. First we can left-multiply by B^{-1}, and obtain

$$\begin{aligned} B^{-1}BX^{-1}A &= B^{-1}AB \\ IX^{-1}A &= B^{-1}AB \\ X^{-1}A &= B^{-1}AB. \end{aligned}$$

Next we can right-multiply by A^{-1}, yielding

$$\begin{aligned} X^{-1}AA^{-1} &= B^{-1}ABA^{-1} \\ X^{-1}I &= B^{-1}ABA^{-1} \\ X^{-1} &= B^{-1}ABA^{-1}. \end{aligned}$$

Finally,

$$\begin{aligned} X &= (B^{-1}ABA^{-1})^{-1} \\ &= (A^{-1})^{-1}B^{-1}A^{-1}(B^{-1})^{-1} \\ &= AB^{-1}A^{-1}B. \end{aligned}$$

[3] 6. We need to show that the product of A^T and $(A^{-1})^T$ is the identity matrix I. We can use the fact that $(XY)^T = Y^T X^T$, so we can write

$$\begin{aligned} A^T(A^{-1})^T &= (A^{-1}A)^T \\ &= I^T \\ &= I. \end{aligned}$$

Thus $(A^{-1})^T$ is the inverse of A^T.