MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 3 | **MATHEMATICS 2050** | WINTER 2026

SOLUTIONS

[4] 1. (a) First we need to identify a point Q on ℓ, such as $(2, 2, -3)$. Next we construct the vector $\mathbf{u} = \overrightarrow{QP} = \begin{bmatrix} -2 \\ 2 \\ 2 \end{bmatrix}$. The projection of $\mathbf{u}$ onto ℓ is

$$\mathbf{p} = \operatorname{proj}_{\mathbf{d}} \mathbf{u} = \frac{\mathbf{u} \cdot \mathbf{d}}{\mathbf{d} \cdot \mathbf{d}} \mathbf{d} = \frac{-6 + 10 + 0}{9 + 25 + 0} \begin{bmatrix} 3 \\ 5 \\ 0 \end{bmatrix} = \frac{4}{34} \begin{bmatrix} 3 \\ 5 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{6}{17} \\ \frac{10}{17} \\ 0 \end{bmatrix}.$$

The vector which represents the straight-line distance from P to ℓ is

$$\mathbf{u} - \mathbf{p} = \begin{bmatrix} -\frac{40}{17} \\ \frac{24}{17} \\ 2 \end{bmatrix},$$

whose norm is

$$\|\mathbf{u} - \mathbf{p}\| = \sqrt{\left(-\frac{40}{17}\right)^2 + \left(\frac{24}{17}\right)^2 + 2^2} = \frac{14\sqrt{17}}{17}.$$

This is the distance from P to ℓ.

[2] (b) The point on ℓ which is closest to P is the beginning point of the vector $\mathbf{u} - \mathbf{p}$. In other words, if this point has coordinates (x, y, z), we want

$$\begin{bmatrix} -\frac{40}{17} \\ \frac{24}{17} \\ 2 \end{bmatrix} = \begin{bmatrix} 0 - x \\ 4 - y \\ -1 - z \end{bmatrix},$$

and so $x = \frac{40}{17}$, $y = \frac{44}{17}$, $z = -3$. Hence the point is $\left(\frac{40}{17}, \frac{44}{17}, -3\right)$.

[2] 2. (a) The plane π and the line ℓ will be parallel if the normal $\mathbf{n} = \begin{bmatrix} -3 \\ 1 \\ -4 \end{bmatrix}$ to π is orthogonal to the direction vector $\mathbf{d} = \begin{bmatrix} 1 \\ -5 \\ -2 \end{bmatrix}$ of ℓ. Observe that

$$\mathbf{n} \cdot \mathbf{d} = -3 - 5 + 8 = 0,$$

so these vectors must be orthogonal. Thus π and ℓ are parallel.

[4] (b) Since π and ℓ are parallel, they are always the same distance apart. Thus we can choose any point P which lies on ℓ and find its distance to π. Such a point is $(-4, 7, 3)$. Now we need any point Q that lies in π, such as $(0, 1, 0)$. Next we construct the vector

$$\mathbf{u} = \overrightarrow{PQ} = \begin{bmatrix} 4 \\ -6 \\ -3 \end{bmatrix}.$$

We want the vector $\mathbf{p}$ which is the projection of $\mathbf{u}$ onto $\mathbf{n}$:

$$\mathbf{p} = \operatorname{proj}_{\mathbf{n}} \mathbf{u} = \frac{\mathbf{u} \cdot \mathbf{n}}{\mathbf{n} \cdot \mathbf{n}} \mathbf{n} = \frac{-12 - 6 + 12}{9 + 1 + 16} \begin{bmatrix} -3 \\ 1 \\ -4 \end{bmatrix} = \frac{-6}{26} \begin{bmatrix} -3 \\ 1 \\ -4 \end{bmatrix} = \begin{bmatrix} \frac{9}{13} \\ -\frac{3}{13} \\ \frac{12}{13} \end{bmatrix}.$$

The distance from P to π (and therefore the distance from ℓ to π) is the norm of this vector:

$$\|\underline{p}\| = \sqrt{\left(\frac{9}{13}\right)^2 + \left(-\frac{3}{13}\right)^2 + \left(\frac{12}{13}\right)^2} = \frac{3\sqrt{26}}{13}.$$

[4] 3. (a) We set

$$k_1 \begin{bmatrix} 1 \\ -6 \\ -3 \end{bmatrix} + k_2 \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} + k_3 \begin{bmatrix} 0 \\ -8 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Thus we have the system of equations

$$\begin{aligned} k_1 + 2k_2 &= 0 \\ -6k_1 + 2k_2 - 8k_3 &= 0 \\ -3k_1 + 2k_2 + k_3 &= 0. \end{aligned}$$

From the first equation, we have $k_1 = -2k_2$. Substituting this into the third equation, we have

$$6k_2 + 2k_2 + k_3 = 0 \quad \Longrightarrow \quad k_3 = -8k_2.$$

Substituting both expressions into the second equation, we obtain

$$12k_2 + 2k_2 + 64k_2 = 0 \quad \Longrightarrow \quad 78k_2 = 0,$$

and therefore $k_2 = 0$. This, in turn, implies that $k_1 = k_3 = 0$ and so only the trivial solution satisfies the system. Thus we conclude that these vectors are linearly independent.

[4] (b) We set

$$k_1 \begin{bmatrix} 2 \\ 6 \\ -4 \\ -4 \end{bmatrix} + k_2 \begin{bmatrix} 7 \\ 3 \\ 1 \\ 3 \end{bmatrix} + k_3 \begin{bmatrix} -4 \\ 6 \\ -7 \\ -9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

This results in the system of equations

$$\begin{aligned} 2k_1 + 7k_2 - 4k_3 &= 0 \\ 6k_1 + 3k_2 + 6k_3 &= 0 \\ -4k_1 + k_2 - 7k_3 &= 0 \\ -4k_1 + 3k_2 - 9k_3 &= 0. \end{aligned}$$

One way to solve this system is to begin with the first equation and obtain

$$k_1 = 2k_3 - \frac{7}{2}k_2.$$

Substituting this into the second equation, we have

$$6\left(2k_3 - \frac{7}{2}k_2\right) + 3k_2 + 6k_3 = 0 \quad \Longrightarrow \quad 18k_3 - 18k_2 = 0$$

so $k_2 = k_3$ and therefore

$$k_1 = 2k_3 - \frac{7}{2}k_3 = -\frac{3}{2}k_3.$$

Next, substituting both of these expressions into the third equation yields

$$-4\left(-\frac{3}{2}k_3\right) + k_3 - 7k_3 = 0 \quad \Longrightarrow \quad 0 = 0,$$

and so any solution with $k_1 = -\frac{3}{2}k_3$ and $k_2 = k_3$ will satisfy this equation. Similarly, substitution into the fourth equation gives us

$$-4\left(-\frac{3}{2}k_3\right) + 3k_3 - 9k_3 = 0 \quad \Longrightarrow \quad 0 = 0.$$

Thus there are many non-trivial solutions to the system (such as $k_1 = -3$, $k_2 = 2$, $k_3 = 2$) and so these vectors are linearly dependent.