

MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 2.3

Math 2050 Worksheet

WINTER 2026

SOLUTIONS

1. (a) The augmented matrix is

$$\begin{aligned}
 & \left[\begin{array}{ccc|c} 1 & 1 & 6 & -5 \\ 4 & -2 & -3 & 1 \\ -2 & 1 & 2 & 0 \end{array} \right] \xrightarrow{\substack{R_2 \rightarrow R_2 - 4R_1 \\ R_3 \rightarrow R_3 - (-2)R_1}} \left[\begin{array}{ccc|c} 1 & 1 & 6 & -5 \\ 0 & -6 & -27 & 21 \\ 0 & 3 & 14 & -10 \end{array} \right] \\
 & \xrightarrow{R_2 \rightarrow -\frac{1}{6}R_2} \left[\begin{array}{ccc|c} 1 & 1 & 6 & -5 \\ 0 & 1 & \frac{9}{2} & -\frac{7}{2} \\ 0 & 3 & 14 & -10 \end{array} \right] \xrightarrow{R_3 \rightarrow R_3 - 3R_2} \left[\begin{array}{ccc|c} 1 & 1 & 6 & -5 \\ 0 & 1 & \frac{9}{2} & -\frac{7}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{array} \right] \\
 & \xrightarrow{R_3 \rightarrow 2R_3} \left[\begin{array}{ccc|c} 1 & 1 & 6 & -5 \\ 0 & 1 & \frac{9}{2} & -\frac{7}{2} \\ 0 & 0 & 1 & 1 \end{array} \right]
 \end{aligned}$$

so then we see that

$$z = 1$$

$$y = -\frac{7}{2} - \frac{9}{2}z = -\frac{7}{2} - \frac{9}{2}(1) = -8$$

$$x = -5 - 6z - y = -5 - 6(1) - (-8) = -3.$$

Thus the solution is $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ -8 \\ 1 \end{bmatrix}.$

(b) The augmented matrix is

$$\begin{aligned}
 & \left[\begin{array}{ccc|c} 1 & -1 & 4 & 5 \\ -1 & 3 & -6 & -13 \\ 1 & 1 & 2 & -3 \end{array} \right] \xrightarrow{\substack{R_2 \rightarrow R_2 - (-1)R_1 \\ R_3 \rightarrow R_3 - R_1}} \left[\begin{array}{ccc|c} 1 & -1 & 4 & 5 \\ 0 & 2 & -2 & -8 \\ 0 & 2 & -2 & -8 \end{array} \right] \\
 & \xrightarrow{R_2 \rightarrow \frac{1}{2}R_2} \left[\begin{array}{ccc|c} 1 & -1 & 4 & 5 \\ 0 & 1 & -1 & -4 \\ 0 & 2 & -2 & -8 \end{array} \right] \xrightarrow{R_3 \rightarrow R_3 - 2R_2} \left[\begin{array}{ccc|c} 1 & -1 & 4 & 5 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & 0 & 0 \end{array} \right].
 \end{aligned}$$

Hence x_3 is not determined, and we let

$$x_3 = t$$

$$x_2 = -4 + x_3 = -4 + t$$

$$x_1 = 5 - 4x_3 + x_2 = 5 - 4t + (-4 + t) = 1 - 3t$$

so the solution is

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 - 3t \\ -4 + t \\ t \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \\ 0 \end{bmatrix} + t \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}.$$

This system has an infinite number of solutions.

(c) The augmented matrix is

$$\begin{aligned} & \left[\begin{array}{ccc|c} 2 & 6 & -4 & 4 \\ -3 & -4 & 1 & -6 \\ -1 & 2 & -3 & 2 \end{array} \right] \xrightarrow{R_1 \rightarrow \frac{1}{2}R_1} \left[\begin{array}{ccc|c} 1 & 3 & -2 & 2 \\ -3 & -4 & 1 & -6 \\ -1 & 2 & -3 & 2 \end{array} \right] \\ & \xrightarrow{\substack{R_2 \rightarrow R_2 - (-3)R_1 \\ R_3 \rightarrow R_3 - (-1)R_1}} \left[\begin{array}{ccc|c} 1 & 3 & -2 & 2 \\ 0 & 5 & -5 & 0 \\ 0 & 5 & -5 & 4 \end{array} \right] \xrightarrow{R_2 \rightarrow \frac{1}{5}R_2} \left[\begin{array}{ccc|c} 1 & 3 & -2 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 5 & -5 & 4 \end{array} \right] \\ & \xrightarrow{R_3 \rightarrow R_3 - 5R_2} \left[\begin{array}{ccc|c} 1 & 3 & -2 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 4 \end{array} \right] \end{aligned}$$

so since the final line implies that $0 = 4$, there is no solution to the system and hence it is **inconsistent**.

(d) The augmented matrix is

$$\begin{aligned} & \left[\begin{array}{cccc|c} 1 & 2 & 0 & -1 & 8 \\ 3 & 6 & 1 & -2 & 23 \\ 0 & 0 & 1 & 1 & -1 \end{array} \right] \xrightarrow{R_2 \rightarrow R_2 - 3R_1} \left[\begin{array}{cccc|c} 1 & 2 & 0 & -1 & 8 \\ 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 1 & -1 \end{array} \right] \\ & \xrightarrow{R_3 \rightarrow R_3 - R_2} \left[\begin{array}{cccc|c} 1 & 2 & 0 & -1 & 8 \\ 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]. \end{aligned}$$

So, first, we see that x_4 is not determined and hence we let $x_4 = t$. Then

$$x_3 = -1 - x_4 = -1 - t.$$

Similarly, x_2 is not determined so we let $x_2 = s$. Finally,

$$x_1 = 8 + x_4 - 2x_2 = 8 + t - 2s.$$

So the solution is

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 8 + t - 2s \\ s \\ -1 - t \\ t \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \\ -1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} + s \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}.$$

This system has an infinite number of solutions.

(e) The augmented matrix is

$$\begin{array}{c}
 \left[\begin{array}{cccc|c} 3 & 1 & -5 & 2 & 2 \\ 0 & -2 & 1 & -1 & -10 \\ 1 & 4 & -4 & 8 & 8 \\ -3 & -1 & 7 & 0 & -6 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{cccc|c} 1 & 4 & -4 & 8 & 8 \\ 0 & -2 & 1 & -1 & -10 \\ 3 & 1 & -5 & 2 & 2 \\ -3 & -1 & 7 & 0 & -6 \end{array} \right] \\
 \\
 \begin{array}{c} R_3 \rightarrow R_3 - 3R_1 \\ R_4 \rightarrow R_4 - (-3)R_1 \end{array} \xrightarrow{\quad} \left[\begin{array}{cccc|c} 1 & 4 & -4 & 8 & 8 \\ 0 & -2 & 1 & -1 & -10 \\ 0 & -11 & 7 & -22 & -22 \\ 0 & 11 & -5 & 24 & 18 \end{array} \right] \xrightarrow{R_2 \rightarrow -\frac{1}{2}R_2} \left[\begin{array}{cccc|c} 1 & 4 & -4 & 8 & 8 \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} & 5 \\ 0 & -11 & 7 & -22 & -22 \\ 0 & 11 & -5 & 24 & 18 \end{array} \right] \\
 \\
 \begin{array}{c} R_3 \rightarrow R_3 - (-11)R_2 \\ R_4 \rightarrow R_4 - 11R_2 \end{array} \xrightarrow{\quad} \left[\begin{array}{cccc|c} 1 & 4 & -4 & 8 & 8 \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} & 5 \\ 0 & 0 & \frac{3}{2} & -\frac{33}{2} & 33 \\ 0 & 0 & \frac{1}{2} & \frac{37}{2} & -37 \end{array} \right] \xrightarrow{R_3 \rightarrow \frac{2}{3}R_3} \left[\begin{array}{cccc|c} 1 & 4 & -4 & 8 & 8 \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} & 5 \\ 0 & 0 & 1 & -11 & 22 \\ 0 & 0 & \frac{1}{2} & \frac{37}{2} & -37 \end{array} \right] \\
 \\
 \begin{array}{c} R_4 \rightarrow R_4 - \frac{1}{2}R_3 \end{array} \xrightarrow{\quad} \left[\begin{array}{cccc|c} 1 & 4 & -4 & 8 & 8 \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} & 5 \\ 0 & 0 & 1 & -11 & 22 \\ 0 & 0 & 0 & 24 & -48 \end{array} \right] \xrightarrow{R_4 \rightarrow \frac{1}{24}R_4} \left[\begin{array}{cccc|c} 1 & 4 & -4 & 8 & 8 \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} & 5 \\ 0 & 0 & 1 & -11 & 22 \\ 0 & 0 & 0 & 1 & -2 \end{array} \right]
 \end{array}$$

so we see that

$$\begin{aligned}
 x_4 &= -2 \\
 x_3 &= 22 + 11x_4 = 22 + 11(-2) = 0 \\
 x_2 &= 5 - \frac{1}{2}x_4 + \frac{1}{2}x_3 = 5 - \frac{1}{2}(-2) + 0 = 6 \\
 x_1 &= 8 - 8x_4 + 4x_3 - 4x_2 = 8 - 8(-2) + 0 - 4(6) = 0
 \end{aligned}$$

and hence the unique solution to the system is

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ 0 \\ -2 \end{bmatrix}.$$

2. The augmented matrix is

$$\begin{array}{c}
 \left[\begin{array}{ccc|c} 1 & -3 & -5 & p \\ 4 & -10 & -16 & 0 \\ 0 & -4 & -8 & q \end{array} \right] \xrightarrow{R_2 \rightarrow R_2 - 4R_1} \left[\begin{array}{ccc|c} 1 & -3 & -5 & p \\ 0 & 2 & 4 & -4p \\ 0 & -4 & -8 & q \end{array} \right] \\
 \\
 \begin{array}{c} R_2 \rightarrow \frac{1}{2}R_2 \\ R_3 \rightarrow R_3 - (-4)R_2 \end{array} \xrightarrow{\quad} \left[\begin{array}{ccc|c} 1 & -3 & -5 & p \\ 0 & 1 & 2 & -2p \\ 0 & 0 & 0 & q - 8p \end{array} \right].
 \end{array}$$

Now we see that the system will be inconsistent if the final row implies $0 = q - 8p$ with $q - 8p \neq 0$. In other words, the desired condition is $q \neq 8p$.

3. A polynomial of degree 2 has the general form $p(x) = ax^2 + bx + c$. We know that $p(1) = a + b + c = -4$, $p(-1) = a - b + c = -12$, and $p(4) = 16a + 4b + c = -7$. Writing these three equations as a system (with unknowns a , b and c) yields the augmented matrix

$$\begin{array}{ccc}
 \left[\begin{array}{ccc|c} 1 & 1 & 1 & -4 \\ 1 & -1 & 1 & -12 \\ 16 & 4 & 1 & -7 \end{array} \right] & \xrightarrow{\substack{R_2 \rightarrow R_2 - R_1 \\ R_3 - 16R_1}} & \left[\begin{array}{ccc|c} 1 & 1 & 1 & -4 \\ 0 & -2 & 0 & -8 \\ 0 & -12 & -15 & 57 \end{array} \right] \\
 \xrightarrow{R_2 \rightarrow -\frac{1}{2}R_2} & \left[\begin{array}{ccc|c} 1 & 1 & 1 & -4 \\ 0 & 1 & 0 & 4 \\ 0 & -12 & -15 & 57 \end{array} \right] & \xrightarrow{R_3 - (-12)R_2} & \left[\begin{array}{ccc|c} 1 & 1 & 1 & -4 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & -15 & 105 \end{array} \right] \\
 & & \xrightarrow{R_3 \rightarrow -\frac{1}{15}R_3} & \left[\begin{array}{ccc|c} 1 & 1 & 1 & -4 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -7 \end{array} \right]
 \end{array}$$

and so we have that

$$c = -7$$

$$b = 4$$

$$a = -4 - c - b = -4 - (-7) - 4 = -1.$$

Hence the polynomial is $p(x) = -x^2 + 4x - 7$.