MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 1 **MATHEMATICS 2050** WINTER 2026

SOLUTIONS

[3] 1. The vectors are parallel if

$$\begin{bmatrix} x \\ -3 \\ 7 \end{bmatrix} = k \begin{bmatrix} -4 \\ 2 \\ y \end{bmatrix}$$

for some scalar k. Thus we must have $x = -4k$, $-3 = 2k$ and $7 = ky$. From the second equation, we immediately have $k = -\frac{3}{2}$. Then, from the first equation,

$$x = -4\left(-\frac{3}{2}\right) = 6.$$

Finally, from the third equation,

$$y = \frac{7}{k} = 7\left(-\frac{2}{3}\right) = -\frac{14}{3}.$$

[4] 2. (a) If possible, we wish to express

$$\begin{bmatrix} 3 \\ -3 \\ 2 \end{bmatrix} = a \begin{bmatrix} 0 \\ -9 \\ 3 \end{bmatrix} + b \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + c \begin{bmatrix} -3 \\ 6 \\ 0 \end{bmatrix}.$$

This results in the system of equations

$$\begin{aligned} b - 3c &= 3 \\ -9a - b + 6c &= -3 \\ 3a &= 2. \end{aligned}$$

From the third equation, we immediately have $a = \frac{2}{3}$ so we can rewrite the second equation as

$$-9\left(\frac{2}{3}\right) - b + 6c = -3 \quad \Longrightarrow \quad -b + 6c = 3.$$

Adding this to the first equation, we have $3c = 6$ so $c = 2$, and therefore $b = 9$. Thus $\left[\begin{smallmatrix} 3 \\ -3 \\ 2 \end{smallmatrix}\right]$ is a linear combination of the vectors, and can be written

$$\begin{bmatrix} 3 \\ -3 \\ 2 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 0 \\ -9 \\ 3 \end{bmatrix} + 9 \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + 2 \begin{bmatrix} -3 \\ 6 \\ 0 \end{bmatrix}.$$

[4] (b) This time, we want to write

$$\begin{bmatrix} 3 \\ -3 \\ 2 \end{bmatrix} = a \begin{bmatrix} 0 \\ -9 \\ 3 \end{bmatrix} + b \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + c \begin{bmatrix} 5 \\ 1 \\ -2 \end{bmatrix}.$$

This results in the system of equations

$$\begin{aligned} b + 5c &= 3 \\ -9a - b + c &= -3 \\ 3a - 2c &= 2. \end{aligned}$$

From the first equation, we have $b = 3 - 5c$. From the third equation, we have $a = \frac{2}{3} + \frac{2}{3}c$. Substituting both of these into the second equation, we obtain

$$\begin{aligned} -9\left(\frac{2}{3} + \frac{2}{3}c\right) - (3 - 5c) + c &= -3 \\ -6 - 6c - 3 + 5c + c &= -3 \\ -9 &= -3, \end{aligned}$$

which is impossible. Thus there are no solutions to this system of equations, and therefore no such scalars a, b and c. In other words, $\left[\begin{smallmatrix} 3 \\ -3 \\ 2 \end{smallmatrix}\right]$ is not a linear combination of the given vectors.

[3] 3. (a) Note that

$$\|\mathbf{u}\| = \sqrt{2^2 + (-6)^2 + (-3)^2} = \sqrt{49} = 7.$$

Thus a unit vector in the same direction as $\mathbf{u}$ is

$$\frac{1}{\|\mathbf{u}\|}\mathbf{u} = \frac{1}{7}\begin{bmatrix} 2 \\ -6 \\ -3 \end{bmatrix},$$

and a unit vector in the opposite direction to $\mathbf{u}$ is

$$-\frac{1}{7}\begin{bmatrix} 2 \\ -6 \\ -3 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} -2 \\ 6 \\ 3 \end{bmatrix}.$$

[3] (b) We set

$$\begin{aligned} \mathbf{u} \cdot \mathbf{v} &= 0 \\ 2x^2 - 6(3x) - 3(-1) &= 0 \\ 2x^2 - 18x + 3 &= 0 \end{aligned}$$

so, by the quadratic formula, $\mathbf{u}$ and $\mathbf{v}$ are orthogonal when $x = \frac{9 \pm 5\sqrt{3}}{2}$.

[3] 4. Expanding the given expression, we can write

$$\begin{aligned}(\mathbf{u}+4\mathbf{v})\cdot(\mathbf{v}-\mathbf{u}) &= \mathbf{u}\cdot\mathbf{v}+4\mathbf{v}\cdot\mathbf{v}-\mathbf{u}\cdot\mathbf{u}-4\mathbf{v}\cdot\mathbf{u} \\ &= \mathbf{u}\cdot\mathbf{v}+4\|\mathbf{v}\|^2-\|\mathbf{u}\|^2-4\mathbf{u}\cdot\mathbf{v} \\ &= -3\mathbf{u}\cdot\mathbf{v}+4\|\mathbf{v}\|^2-\|\mathbf{u}\|^2 \\ &= -3(-2)+4(1^2)-3^2 \\ &= 1.\end{aligned}$$